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Past-Present-Future: Everlasting Challenges of Geodesy

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Abstract

Geodesy is one of the oldest disciplines of natural sciences. It was developed by practical needs of surveyors to measure and divide the land and also for the basis of mapping. Very soon techniques and methods developed for these very practical tasks were applied also for much wider purposes. Efforts to measure the precise size and shape of the Earth date back to ancient times but before the space age, there was no means to do such measurements accurately on a global level. Vast distances, lack of suitable technique or mathematical tools, and finally oceans separating the continents formed insurmountable barriers. Always there have been practical demands or theoretical ambitions which were not possible to solve at that time. Sometimes it took even hundreds of years to find a solution. In this presentation I describe with a few examples the everlasting race of requirements, needs and technical possibilities, leading to technological development and new observations which, in turn, sets new demands for geodetic information. There are also examples of how erroneous observations may affect even our worldview.

Keywords

historical geodesy, size of Earth, longitude, triangulation, satellite geodesy

1 Size of the Earth

Although there were some earlier attempts to estimate the size of the Earth, the first well-known determination is usually credited to Eratosthenes of Cyrene around 240 BC. He determined the circumference of the Earth with remarkable accuracy using the observations of the Sun at two sites on the summer solstice.

Eratosthenes had heard that on the summer solstice the Sun shone to the bottom of a deep well in Syene (now the city of Aswan in Egypt). At the same time, he measured at Alexandria that the Sun at midday cast a shadow, which was equivalent to 1/50th of a circle, or about seven degrees angle to the zenith. This means that the circumference of the Earth is 360°/7° times the distance of Syene and Alexandria. He assumed that the ground distance from Alexandria to Syene was 5 000 stadia which was based on the measurements by bematists, professionals specialised to measure distances by counting their steps. With these values, Eratosthenes concluded that the circumference of the Earth is 252 000 stadia.

We do not have the original calculation of Eratosthenes, but our knowledge is based on the simplified description of Cleomedes, which dates back to later centuries. First of all, we do not know for sure, what length of the stadion was used in Egypt at the time of Eratosthenes, but the most probable one is 157.7 metres. This gives the circumference of the Earth 39 700 km which is amazingly close to the actual value of 40 000 km.

There are also sources of error or uncertainties that will affect the measurements. Syene is not directly south of Alexandria, neither it is not on the Tropic of Cancer so the Sun was not exactly at zenith on the high noon of Summer Solstice. As well there is uncertainty on the distance between the cities. The excellent result is partly due to good luck and errors cancelling each other, but it also shows that the method itself was adequate.

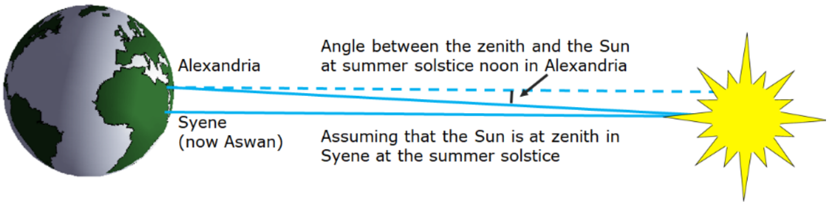

Figure 1: Method of Eratosthenes to measure the circumference of the Earth.

Unfortunately, Eratosthenes' result was partly forgotten or abandoned during the next centuries, and later determinations about the size of the Earth were less accurate. For instance, Poseidonius (135–51 B.C.) made a similar determination. Using the meridian arc from Alexandria to Rhodes he observed that the star Canopus was just rising above the horizon in Rhodes while in Alexandria the elevation angle was 7.5° at the southern culmination. Poseidonius got for the circumference of Earth 240 000 stadia.

His method is not accurate because the elevation angle of objects near the horizon is affected by the refraction of the atmosphere. The elevation of Canopus in Alexandria was also in error. It is in reality 5.25°. This badly erroneous value was not changed, but when the distance between Rhodes and Alexandria was later fixed to a more correct value, one obtained the circumference of the Earth far too small value of 180 000 stadia.

In his book Geographike Hyphegesis (Geography), Ptolemy ended up with a value of the circumference that was 30% too small. Instead of Eratosthenes' value, he adopted later (erroneous) determinations. The value of Ptolemy was used through the Middle Age and possibly affected that Columbus decided to head for the west to find a new and fast route to India and China. Columbus expected that the distance was much shorter than what it was in reality. We will return on this issue with the quest of longitude.

2 Problem of Longitude

Latitude is easy to measure by stellar observations. It needs only to measure the height of the Sun or a star at its highest at two sites to get the latitude differences of the sites. The equator is a natural zero of the latitude. However, there is no logical zero for longitude nor any absolute way to measure the longitude.

The use of the Greenwich meridian as the zero is just a commonly adopted convention. Many sites have been used for this purpose during centuries: Rhodes, Alexandria, Kap Verde or the Canary Islands at the western border of the known world in antiquity, or e.g., Paris, Pulkovo or Washington in modern times. Our habit to use Greenwich as the zero meridian is only a bit more than 100 years old. In 1884, representatives of 25 countries agreed at a conference in Washington, USA, that the Greenwich meridian would be adopted as the zero line.

The zero longitude itself is not the problem, but we must have a method to measure longitude differences, or actually, the difference of local times simultaneously at two sites. The Earth rotates 360° in 23^h 56^m, i.e., 15° in one hour. If the longitude difference between the two sites is 15°, the local time difference is one hour. This can be determined by astronomical observations if we can get a common time somewhere. A stellar event visible simultaneously at both sites can be used as a clock. The Lunar eclipse is suitable for timing, but as history shows, it is not easy.

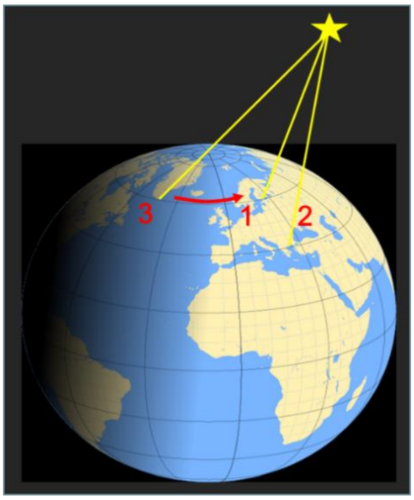

Figure 2: Difference of determination of latitude and longitude. At sites 1 and 2 a star is visible at different elevation because of the latitude difference. It is sufficient to measure the elevation angle when the star is seen at its highest. Sites 2 and 3 are on different longitude, so the star is seen at the same time in different direction. If one can measure the difference of local times when the star is seen e.g. south, then the longitude difference is known.

The problem of longitude remained until the 18th century when the first portable precise clock was built. The difficulty is easily seen by looking at ancient maps. Maps are quite realistic in the North-South direction but there can be huge distortions in East-West. This is the consequence of latitude determination.

One of the best-known early determinations was made using the famous Lunar eclipse which occurred on 20 September 331 B.C., 11 days before the Battle of Gaugamela by Alexander the Great and Darius III. Ptolemy described the eclipse in his book Geography and ended up to a conclusion that the longitude difference between Gaugamela and Cartago is 45°. This was in error by 11 degrees; the actual longitude difference is 34°. The main reason was the inaccurate determination of the local times of the event. One hour uncertainty in the description of the event, e.g. relative to the sunset, makes 15° error in longitude.

As seen in the previous chapter, Ptolemy assumed the Earth much smaller than it is. Therefore, the Mediterranean Sea was believed to reach further East than what it did. Or the opposite, the distance from Europe to China was believed to be relatively short over the Atlantic Ocean. This must be one of the arguments which motivated Columbus for his first voyage in 1492. Further, mixing of Arabic mile (about 1 830 m) and Italian mile (about 1 480 meters), Columbus estimated the distance from the Canary Islands to Japan as 2 400 nautical miles (~3 500 km) which is 1/5 of the actual value!

Luckily, the American continent happened to be there in between but even that was almost twice the distance Columbus originally expected. Columbus was still quite confident that he is not far from Asia. The expeditions made two determinations of longitude on the "East Indian Islands" in 1494 and 1503, both using the Lunar eclipses. The first determination gave the longitude 20° and the second one even 40° too west. A good question is whether these were real uncertainties of observations or did Columbus interpret the imprecise local timings of the eclipses in a way that gave the longitudes more favourable to his belief.

Gradually, understanding of distances and longitudes improved, but the basic problem remained: how to transfer the time overseas. As late as 1682 Louis XIV complained that he loses more land to his astronomers and surveyors than to his enemies. That year an improved map of France was published, and with new longitude determinations, the west coast of France was moved quite considerably to the east.

The final solution and the end of the latitude problem came finally in 1736 when the first marine clock H1 of John Harrison was tested. With the portable clock, the exact time of the home harbour was possible to transfer to the destination. This enabled the precise determination of longitude everywhere.

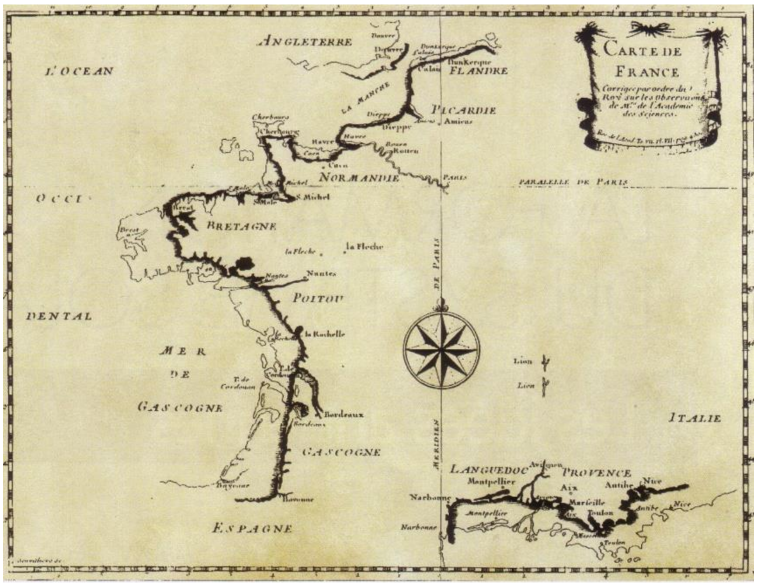

Figure 3: Map of the French coast, corrected by the Academy of Sciences in 1682. Mémoires de l'Académie royale des Sciences, T7. Public domain.

3 Shape of the Earth

To determine the exact shape of the Earth, one needs many technical and theoretical inventions but also laborious geodetic observations utilising the new technique. The inventions made in the 16th and 17th centuries created at the same time the basis and practical need for the development of geodesy. These include the invention of triangulation, theodolite, and mathematical tools to analyse observations, but also the theoretical background to understand the shape of the Earth. The works of Gemma Frisius (1508–1555), Tycho Brahe (1546–1601), and Snellius (Willebrord Snel van Rooyen, 1580–1626) have been revolutionary in geodesy, though much less known to the general public than the revolution in astronomy begun by Copernicus.

Gemma Frisius published the principle of triangulation in 1533 in his supplement Libellus de locorum describendorum ratione to the reprint of Petrus Apianus' (1495–1552) book Cosmographia. The method enabled geodetic measurements and mapping of large areas with unprecedented accuracy. The superiority of triangulation is due to the measurement of the angles of a triangle, which is many times faster than the measurement of distances. Once the angles of the triangles have been measured, the scale of the whole network is obtained if the length of one side of the triangle is known.

Although Frisius has been considered the father of triangulation, distances using triangles were already measured in ancient China. In the West, Chinese measurements were unknown, so there is a good reason to say that triangulation has developed independently in Europe.

One of the first triangulation was made by famous Danish astronomer Tycho Brahe. He made a triangulation at Øresund in 1578–1579 to determine the location of the island of Hven and his observatory, Uraniborg, relative to both shores of the strait. In the east, the triangular network extended to Copenhagen and Malmö, and in the west to Helsingør and Helsingborg.

In 1615 Snellius completed a triangulation measurement in the Netherlands where he had established a triangulation network between Alkmaar and Breda, the distance of which is about 116 km. The motivation of Snellius was to measure precisely the length of the meridian arc which can be used to calculate the circumference of the Earth with an unpreceded accuracy. In later centuries the arc measurement (or grade measurement) became the main technique to measure the shape of the Earth. (The circumference of the Earth Snellius computed from his observations was 38 653 km.)

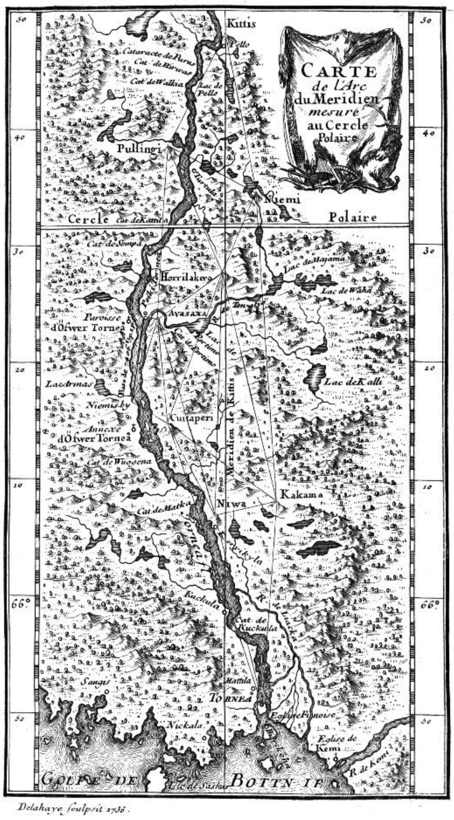

Figure 4: Triangulation network of Maupertuis at the Torne river valley 1736-1737.

After Snellius time there came into use some essential technical inventions which improved the accuracy of measurements. First is the telescope. The invention of the telescope in the early 17th century enabled the magnification of the observed target. Precisely graduated circles to measure angles and hair crosses in the eyepieces combined with the telescope was gradually developed into a theodolite, the basic instrument of geodesy for centuries.

Isaac Newton (1642–1727) in the third edition of Philosophiæ Naturalis Principia Mathematica (1726) ended up depicting the Earth as a flattened ellipsoid. In France, mainly based on the arc measurements near Paris and computations of Jacques Cassini (1677–1756), the results showed that the Earth is prolated, not flattened. To resolve the dispute, the French Academy of Sciences equipped two expeditions for degree measurement: one to the equator, and the second one to the north. Both expeditions had to measure the length of the meridian arc corresponding to one degree by triangulation and make the astronomical determination of latitude. Length of a 1 degree meridian arc is longer near the poles than at the equator if the Earth is flattened.

The expedition of Pierre Louis Moreau de Maupertuis travelled to Lapland to the Torne river valley. The question was solved already in 1736–37 when measurements of the Maupertuis expedition showed that the length of the one-degree-long meridian arc is longer at the north than in France. Some years later the result was confirmed when the Peruvian expedition returned to France.

The value of flattening, 1/123, obtained by Maupertuis was too big. Later on, mainly the reconstructions in the 1920s by Yrjö Leinberg of the Finnish Geodetic Institute showed that most of the error was due to the observing uncertainties and errors of the devices used for the astronomical latitude determination. It was a bit of luck that the errors were affecting in that direction; otherwise, the debate over flattening could have lasted much longer.

There were also many other consequences of the expedition's visit to Lapland. Description of the measurement, La Figure de la Terre, published by Maupertuis in 1738, attracted a large number of readers. Even more popular was the travelogue published in 1744 by the abbot Reginald Outhier (1694–1774), the priest of the Maupertuis expedition. Above all, the romantic nature descriptions attracted the attention and interest of contemporaries towards the distant wonderland of Lapland.

A young talented mathematician Alexis Clairaut (1713–1765) participated in the journey. He made measurements of gravity with a pendulum. In later years, Clairaut developed a mathematical theory that could be used to calculate the flattening of the Earth from gravity observations. At his time, however, there were no statistical methods that could handle a large amount of observational data.

The Finnish interpreter for the expedition, Anders Hellant (1717–1789) later made his dissertation at the University of Uppsala. He returned to Lapland and made his career there. In 1750 he was invited as a member of the Royal Swedish Academy of Sciences. Moreover, the equipment of the Maupertuis expedition was donated to Sweden, and those were used in the coming decades for geodetic and mapping purposes.

Finally, there was one additional consequence, still visible today, namely the length of the metre. During the Great French Revolution of 1793, the system of measures was also reformed. The purpose was to define the length of the meter as 1/10 000 000 of the quarter of the length of the meridian ellipse passing through Paris. The excessive value of the flattening of the Earth, derived from the errors of the degree measurement in France and partly from the results of Maupertuis, caused the metre to become about 0.2 mm too short. Thus, the actual length of the meridian ellipse is 40 008 km and not 40 000 km as intended.

Much more extensive works were conducted in the 19th century. The most famous one was the Russo-Scandinavian grade measurement, or the Struve Arc, 1816-1852. It was conducted by the German-born astronomer Friedrich Georg Wilhelm Struve. The chain of triangles stretches from the Black Sea to Hammerfest in Norway; a total of 2820 km. The value of the measurement is recognized even today, and the Struve Geodetic Arc was inscribed on the UNESCO World Heritage List in 2005.

4. Over the Oceans

Geodetic measurements became more and more accurate as well our information on the planet Earth. There was still one barrier to overcome: how to connect geodetic measurements and networks over oceans. Before the space age possibilities were quite limited.

The famous Swiss mathematician and expert in celestial mechanics, Leonhard Euler (1707–1783), mentioned an idea to use total solar eclipses in distance measurements. When starting times of an eclipse are observed at two sites along the zone of totality, the distance between the sites can be computed if the speed of the shadow on the Earth surface is known. However, it was not possible to perform such a measurement at Euler's time. Two new inventions were needed: the radio and the movie camera. With a radio, one can get precise time signals simultaneously at both sites and the eclipse is recorded with a movie camera together with the time signal markings on the film.

Such a measurement was made successfully in 1947 when expeditions of T.J. Kukkamäki and R.A. Hirvonen of the Finnish Geodetic Institute measured the distance of sites in South America and Africa with an accuracy of 141 m. This was a remarkable increase in accuracy and proof of the potential of space geodetic techniques.

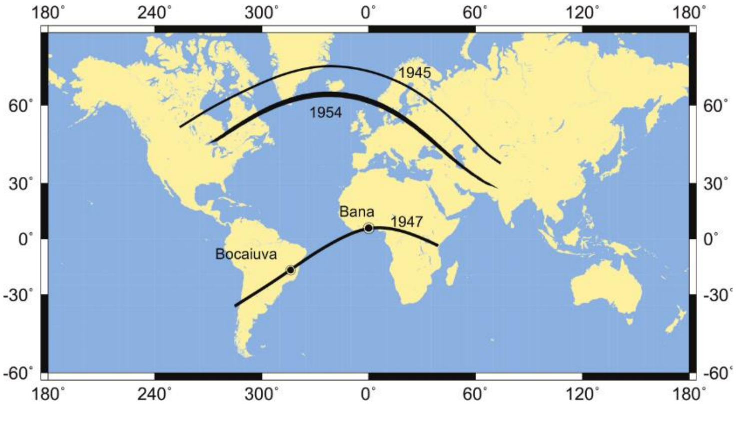

Figure 5: Three total Solar eclipses when the distance measurement was applied. Only the 1947 eclipse measurement was successful.

Solar eclipses are rare and even rarer to have a favourable path of totality to connect two continents. In 1954 a large international campaign was organized to observe the great solar eclipse ranging from Northern USA and Canada to Scandinavia, the Soviet Union, Iran, Pakistan and India. Mother Nature showed her bad side to all expeditions: the eclipsed Sun was behind clouds! This was the last trial to use eclipses for distance measurement. The first artificial satellite Sputnik was launched three years later, in 1957. Finally, the eternal dream of geodesists became true. Satellites enabled measurements between continents.

First positioning experiments were made almost immediately based on optical observations of satellites using stellar triangulation. The principle of stellar triangulation was introduced in Finland much earlier by Academician Yrjö Väisälä. He described the method in 1946 and later on tested an application where a flashlight is lifted with a balloon up to 25 – 30 km above the ground. When the flashes are photographed simultaneously at two sites against the starry background, one can compute the relative positions of the sites. A five-point network was measured by Juhani Kakkuri in Southern Finland in the late 1960s – the early 1970s. The longest distances were more than 200 km.

With satellites, one is not limited to a couple of hundreds of kilometres but can measure distances of even thousands of kilometres thus allowing intercontinental measurements. Especially the USA launched several satellites for this purpose, like Pageos (Passive Geodetic Earth Orbiting Satellite, 1966). It was a 30-m balloon-like satellite that was easy to observe when sunlit. As a result of international cooperation, a global triangle network of 45 points was measured where the mean accuracy of points was better than 5 m.

The weather is a severe problem also with optical observations of satellites. Additionally, measuring the position of a satellite relative to the stars limited the accuracy and there were no means to observe the distance of a satellite. Optical observations were soon replaced by radio waves that can be detected through the clouds.

First Transit Doppler satellites were launched in the late 1960s. The system was primarily meant for navigation at seas, but it was also used for geodetic purposes. Observing passes of a Transit satellite at the same site during several days, one achieved a relative accuracy of a few decimetres in geodetic positioning. The Transit Doppler system was officially ceased in 1996 when it had completely replaced by GPS (Global Positioning System).

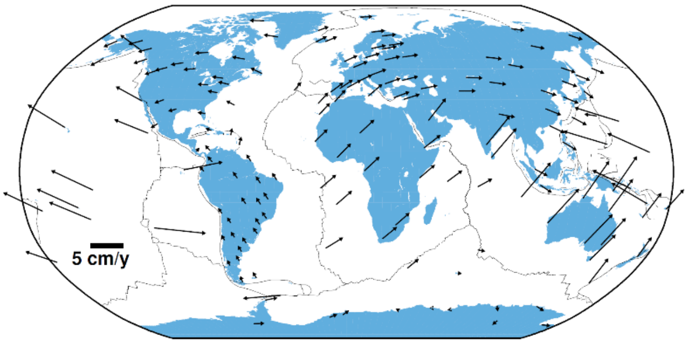

Figure 6: Continents are moving a few centimetres per year. Movements can be measured with global GNSS observations.

GPS allows precise measurements almost in real-time, it is completely weather-independent, and receivers are inexpensive and simple to use. In addition to GPS, there are three other satellite systems. Together these are called GNSS, Global Navigation Satellite Systems. GNSS applications have increased rapidly and even small handheld navigators can use all four satellite systems simultaneously. Today everybody can get 1-m accuracy in minutes, which was merely a dream in the 1960s. With geodetic GNSS receivers, one can get a position within centimetres, measure the global movement of continents or local deformations or even get accurate time anywhere.

5. The Future

The invention of triangulation and theodolite meant a revolutionary leap in geodesy. Even a bigger leap was the beginning of the space age with the first satellites. The first time it became possible to measure the Earth as a single geodetic object over oceans and inhabited deserts. Over the decade measurement accuracy and our understanding of the planet Earth increased a hundredfold, and during the next decades, the speed continued.

Global reference frames and space-born measurements are still getting more and more accurate, allowing many new global research topics like measuring the climate change-induced sea-level rise and melting of ice sheets. Technical development brings us new instruments and new methods to push the limits further and further. The rapid development of satellite positioning and numerous applications has made possible the widespread use of geospatial data.

Modern society is fully dependent on data and products made with space geodetic techniques, albeit most people may not recognize it. This will be one of the challenges of geodesy in the future. How to secure geodetic networks and funding, maintain high-quality and high-level education, and how to disseminate information and products where those are most urgently needed. The International Association of Geodesy and other related entities have already received promising development within the United Nations Subcommittee on Geodesy. This will be our next geodetic challenge.

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