Intl Symposium in honour of
Professor Dr.Ing.habil. Dr.Ing.h.c.mult.(AvH)
Erik W. Grafarend
& AvH 250th Anniversary

Alexander von Humboldt and the Cosmos: The Scientific Work of Erik

Peiliang Xu

Disaster Prevention Research Institute

Kyoto University

Gokasho, Uji, Kyoto 611-0011, Japan

I. Alexander von Humboldt and the COSMOS

II. The scientific work of Erik

I. Alexander von Humboldt and the COSMOS

250 Anniversary of the birth of AvH

14 Sept 1769 – 6 May 1859
Portrait by Joseph Karl Stieler

Two Scientific Expeditions:

1. Spanish American Expedition
5 June 1799 ➔ 1 August 1804

2. Russian Expedition
May ➔ November 1829

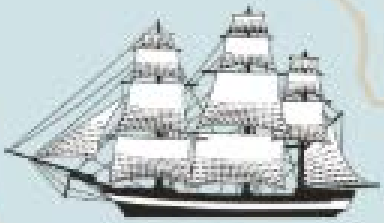
Alexander von Humboldt known as

A. naturalist
B. observer of the Universe
C. curious scientific adventurer
D. bold prophet

➔ Founder of Geographical Science

https://humboldt-heute.de/en/stories/humboldts-travels

50 highly advanced instruments of the era



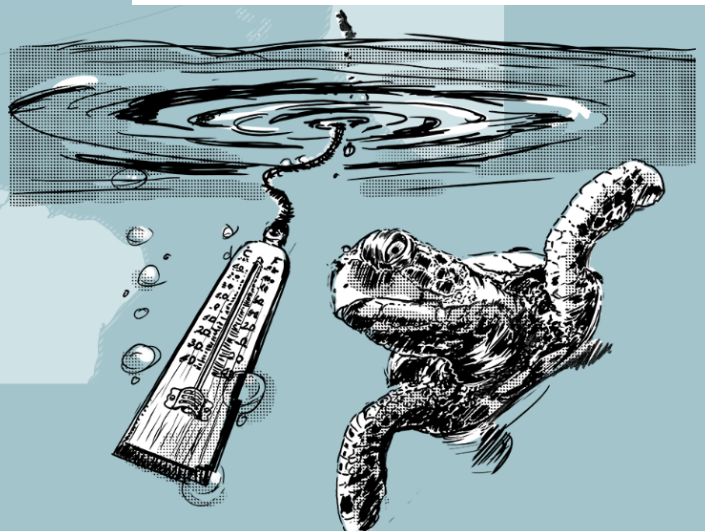
Humboldt crossed the Atlantic aboard the Portuguese corvette Pizarro.

Sextant: measure the angle to the Sun ➔ latitude

chronometer: measure the time difference of the Sun at the highest point ➔ longitude

Dip circle: measure the magnetic intensity

Telescopes

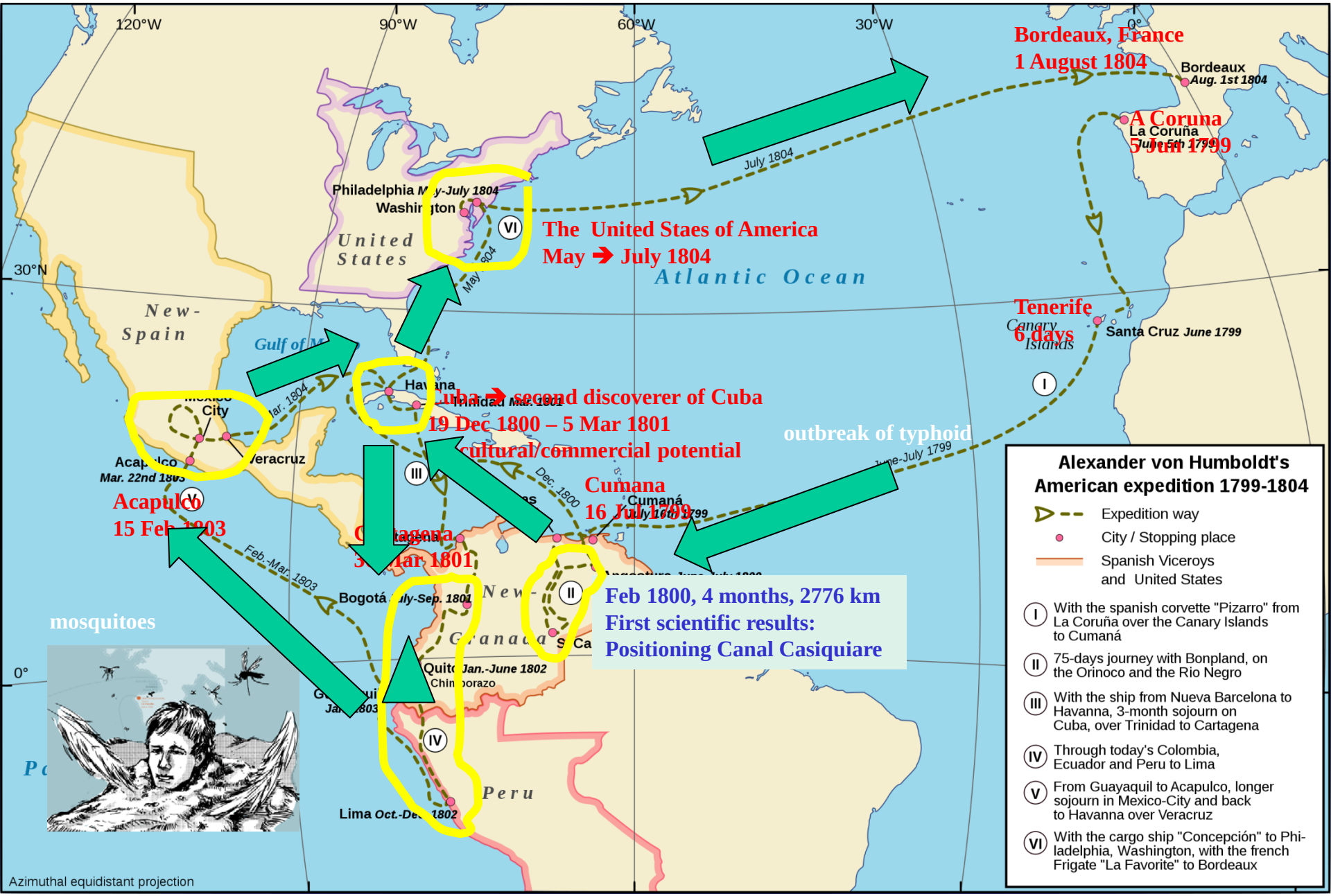


thermometer: measure the temperature of seawater (Pacific)

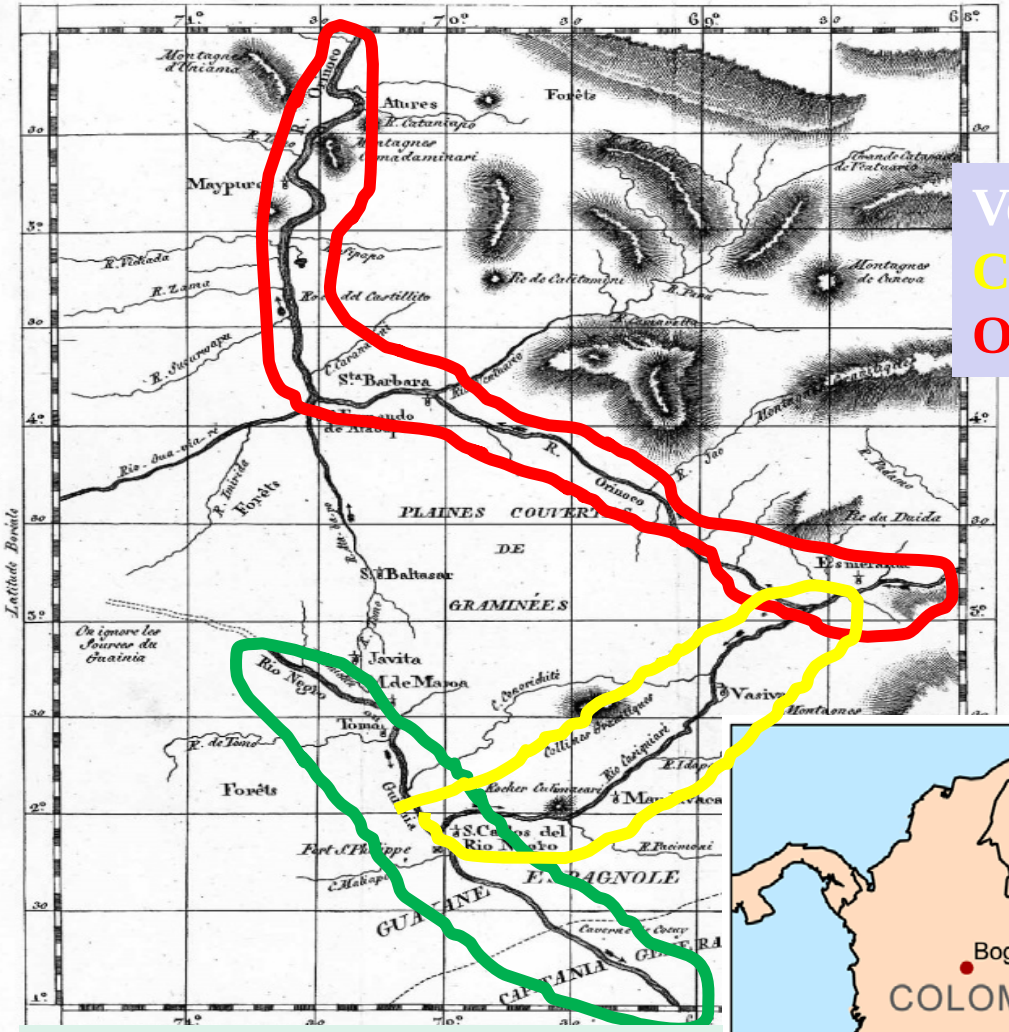
He has more instruments than any previous explorer – sextants, quadrants, telescopes, marine chronometers, barometers, thermometers and all sorts of devices with amazing names such as inclinatorium, declinatorium, cyanometer, eudiometer, hydrometer and hyetometer

en.wikipedia.org/wiki/Alexander_von_Humboldt

https://humboldt-heute.de/en/stories/humboldts-travels

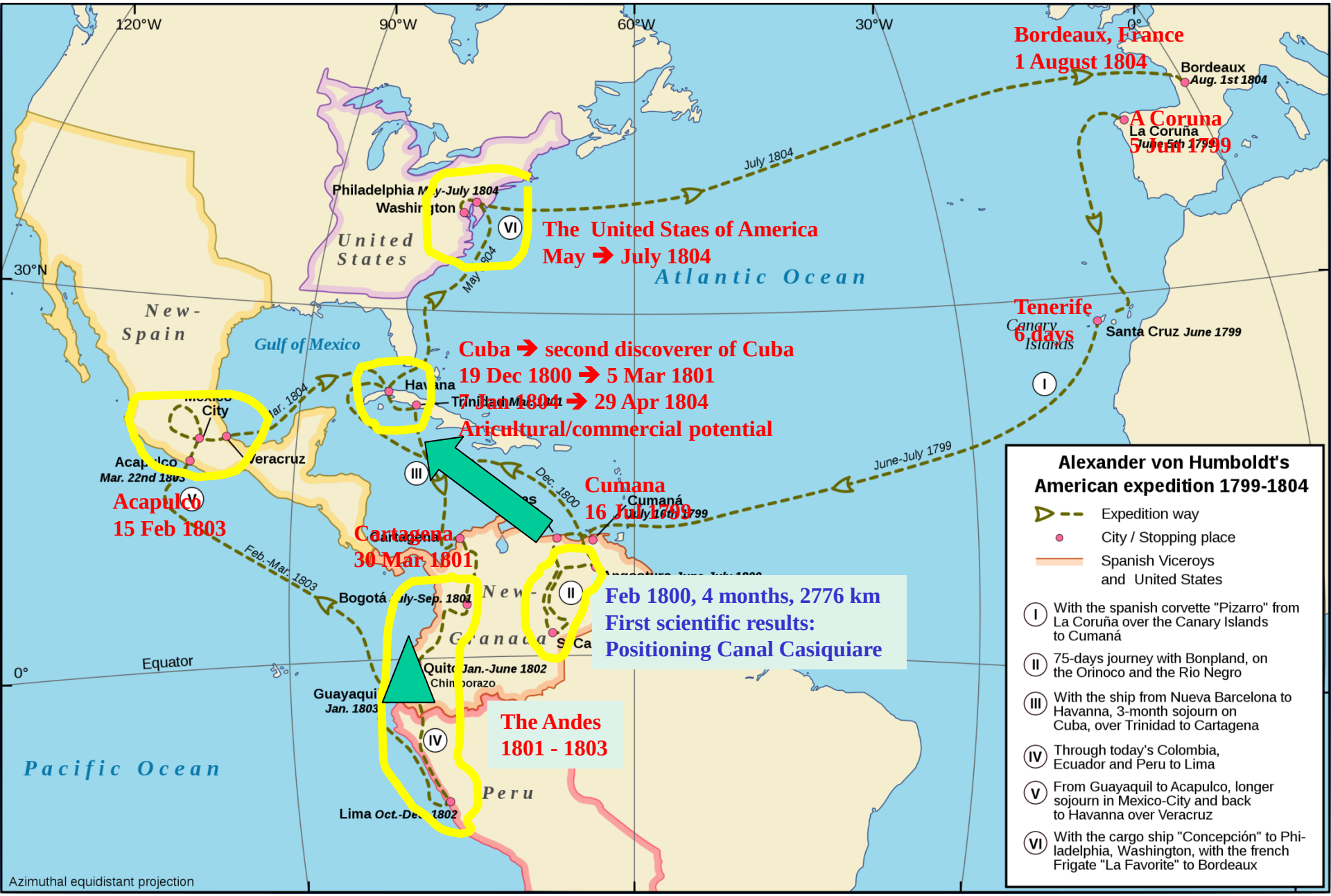


Venezuela 1799➔1800
Canal Casiquiare
Orinoco ←→ Amazon



Feb 1800, 4 months, 2776 km
First scientific results:
Positioning Canal Casiquiare
meteor shower, electric eels
➔ electricity + magnetism

en.wikipedia.org/wiki/Alexander_von_Humboldt



Cuba
19 Dec 1800 ➔ 5 Mar 1801
7 Jan 1804 ➔ 29 Apr 1804

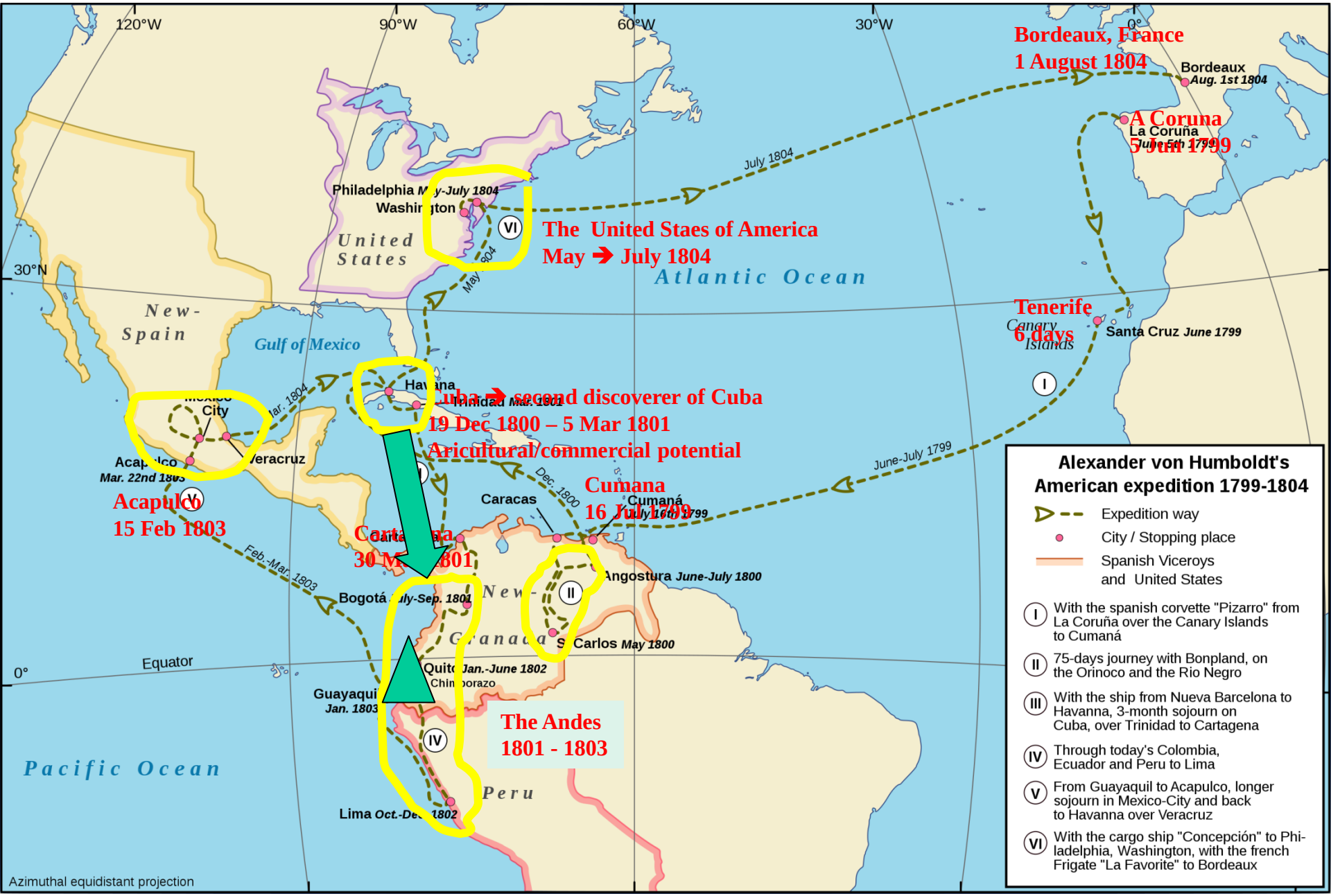
1. **Survey the city;**
2. **Collect statistical information on Cuba's population, production, technology and trade;**
3. **Conduct mineralogical surveys;**
4. **Collect the island's flora/fauna**

➔ agricultural/commercial potential

➔ first work of national geography in history

Botanical drawing by Humboldt

en.wikipedia.org/wiki/Alexander_von_Humboldt



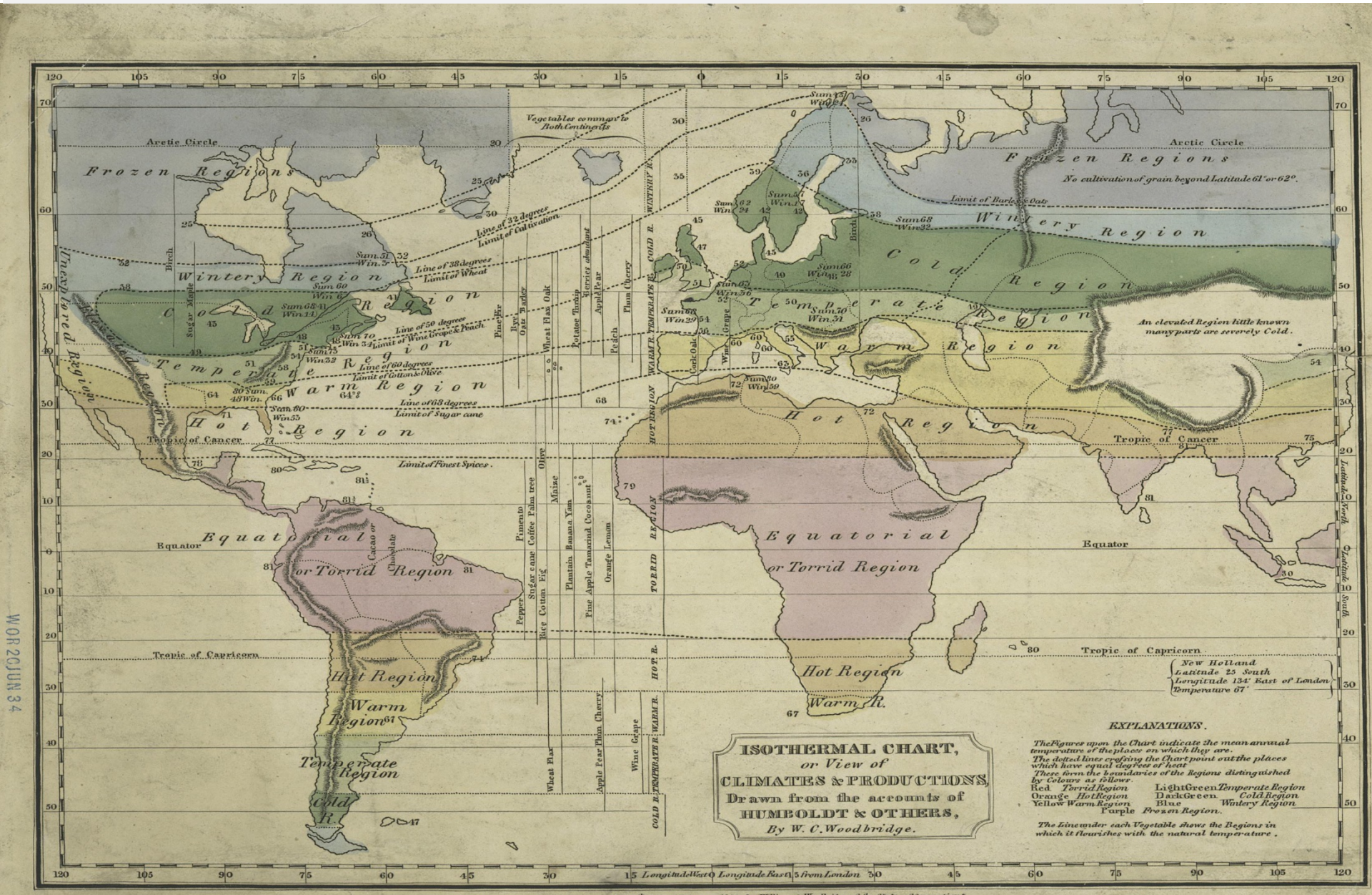
The Andes 1801 – 1803

A. get access to the huge pictorial records of botanies
➔ realizing the importance of fine recordings/images;
B. creating a world record of climbing Chimborazo to 5878 m (about 305 m below summit) ➔ volcanoes;
C. temperature, altitude, humidity, atmosphere pressure, animals & plants at each elevation ➔ environmental science
D. observing the transit of Mercury (1802/11/09)

Illustrations from Spanish botanist
Jose Celestino Mutis in Colombia

Humboldt and Bonpland
near the foot of the Chimborazo volcano
Painting by Friedrich Georg Weitsch (1810)

The first isothermal map in the world



ISOTHERMAL CHART, or View of CLIMATES & PRODUCTIONS, Drawn from the accounts of HUMBOLDT & OTHERS, By W. C. Woodbridge.

EXPLANATIONS.

The Figures upon the Chart indicate the mean annual temperature of the places on which they are. The dotted lines crossing the Chart point out the places which have equal degrees of heat. These form the boundaries of the Regions distinguished by Colours as follows.

Red	Torrid Region	Light Green	Temperate Region
Orange	Hot Region	Dark Green	Cold Region
Yellow	Warm Region	Blue	Wintery Region
Purple	Frozen Region.		

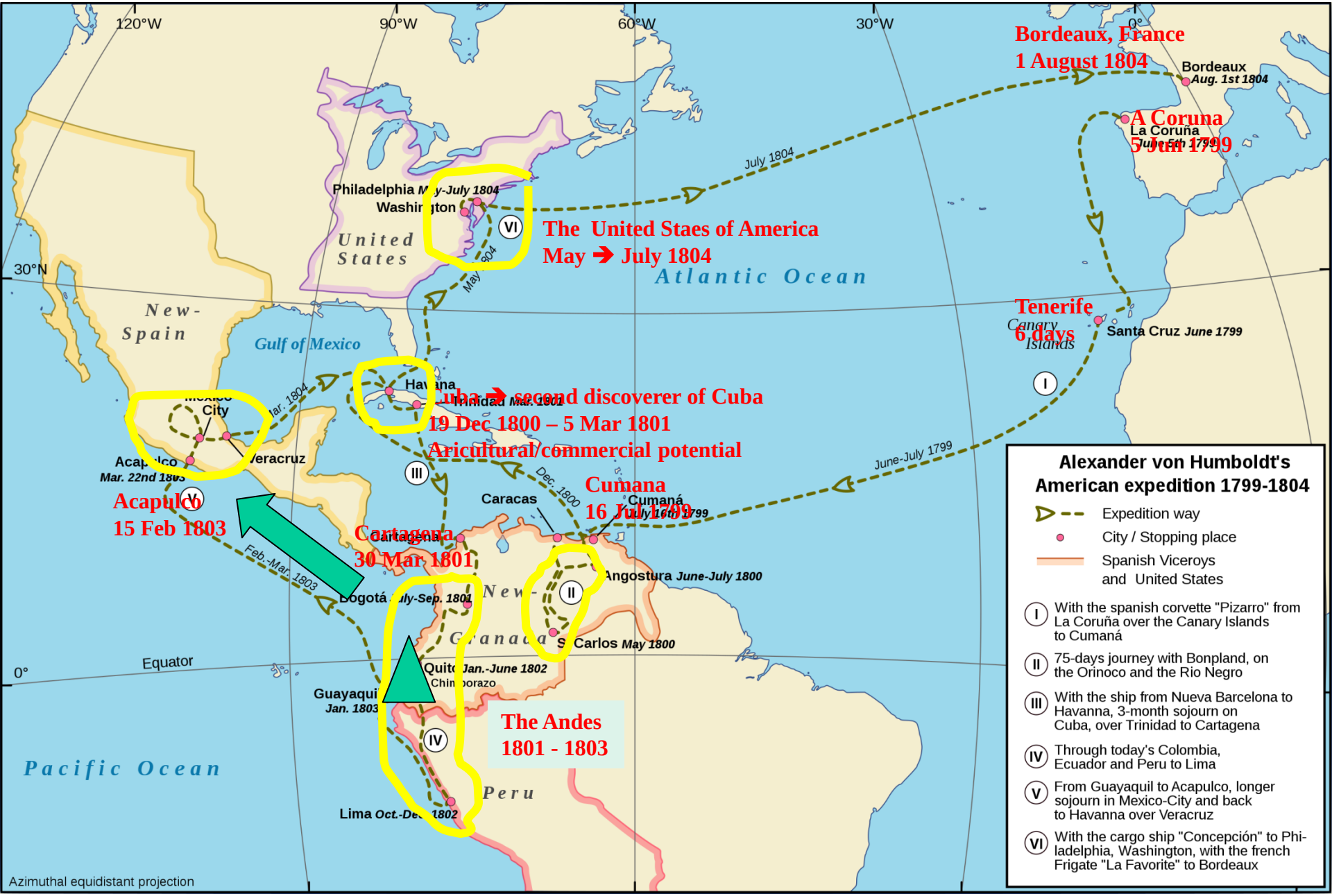
The Line under each Vegetable shows the Regions in which it flourishes with the natural temperature.

Entered according to Act of Congress the 15th day of January 1823 by William C. Woodbridge of the State of Connecticut.

(Map Div. 02-306) World. 1823

WOR 20 JUN 34

en.wikipedia.org/wiki/Alexander_von_Humboldt

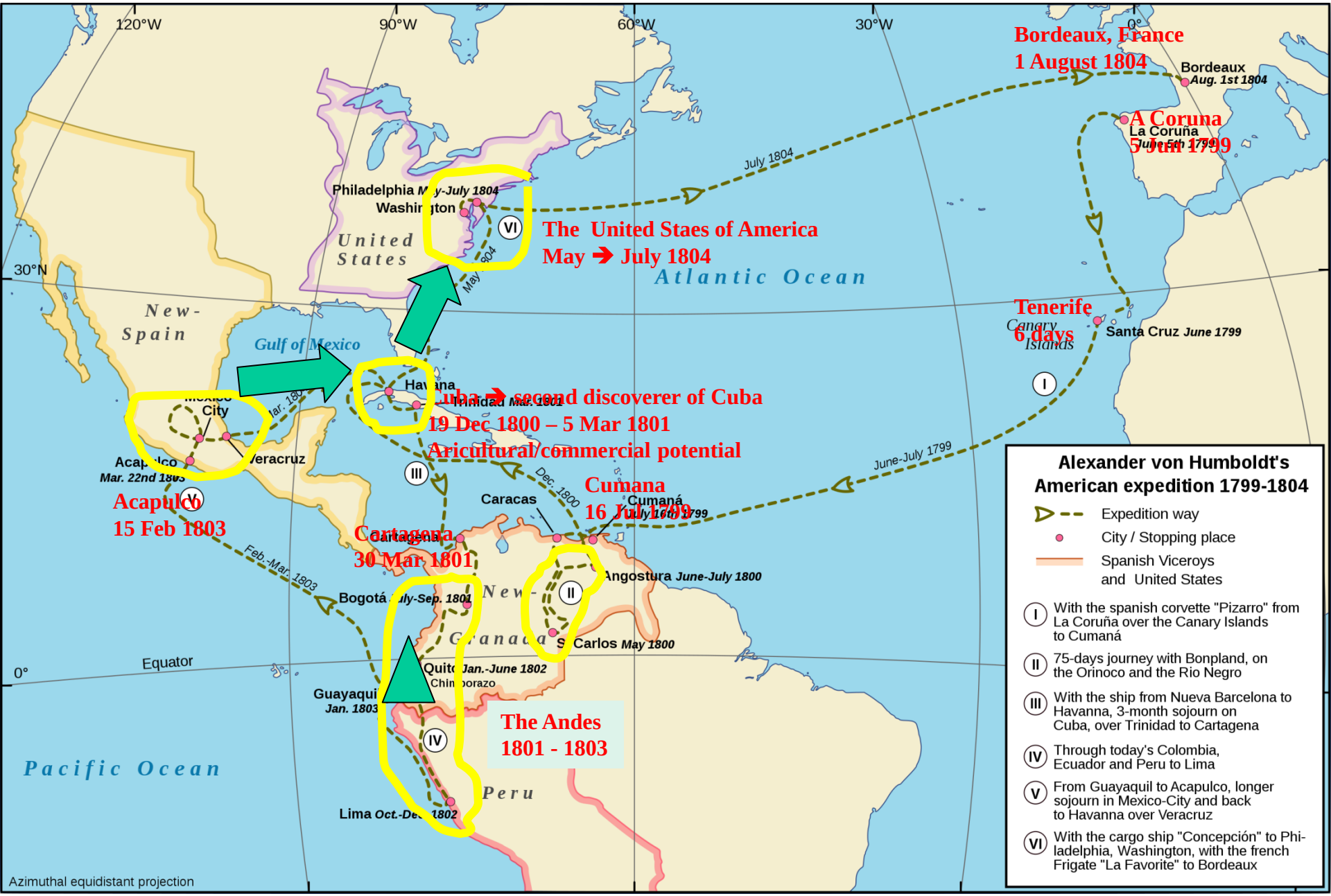


Mexico (New Spain) 1803 – 1804

A. first precise geodetic measurement in Mexico city, incorrect by about 300 miles in position before, and measurements of elevation, the visual depiction of elevation ➔ first contour map?
B. research on mining geology;
C. using graphs and charts to show data

Basalt prisms: Santa Maria Regla Mexico by Humboldt

en.wikipedia.org/wiki/Alexander_von_Humboldt



The United States of America
May ➔ July 1804

scientific diplomacy and meeting with the U.S. President Jefferson and some of the major scientific figures of the time

based on his research of mining geology, he correctly predicted diamonds (Georgia, N Carolina, Virginia), gold deposits and platinum in California
➔ scientific prophet

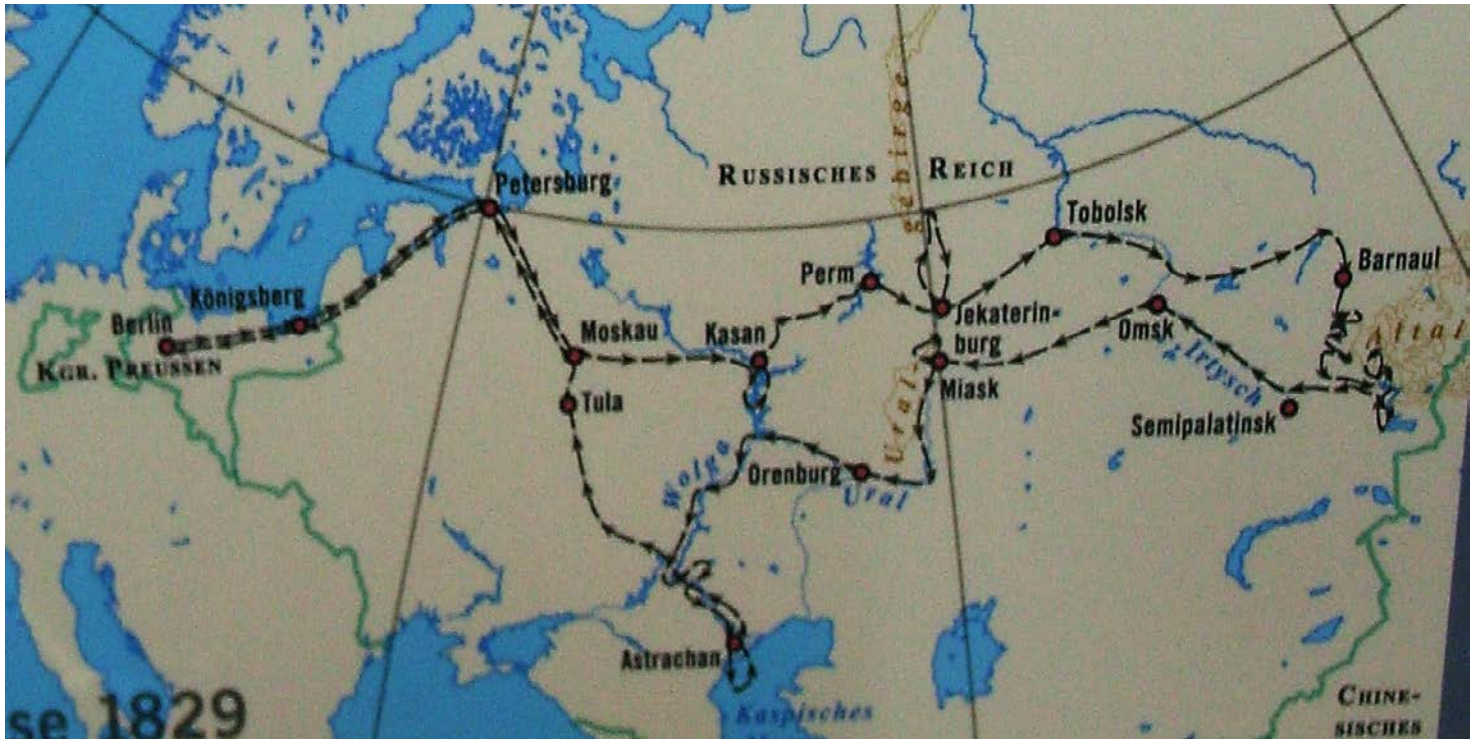
Alexander von Humboldt

➔ **(L Agassiz on the 100th anniversary of the birth of AvH)**

"The scientific discoverer of America"

The Russian Expedition
May ➔ November 1829
twenty-five weeks with a distance of 15,472 km
(sometimes 160 km/day)

- **expedition to the Ural maintains and Siberia;**
- **investigate magnetism of maintains and mineral deposits**



Alexander von Humboldt

- **great in all sciences:**

  geography, geology, geophysics, cartography, climatology, meteorology, mineralogy, botany, anatomy, biology, zoology

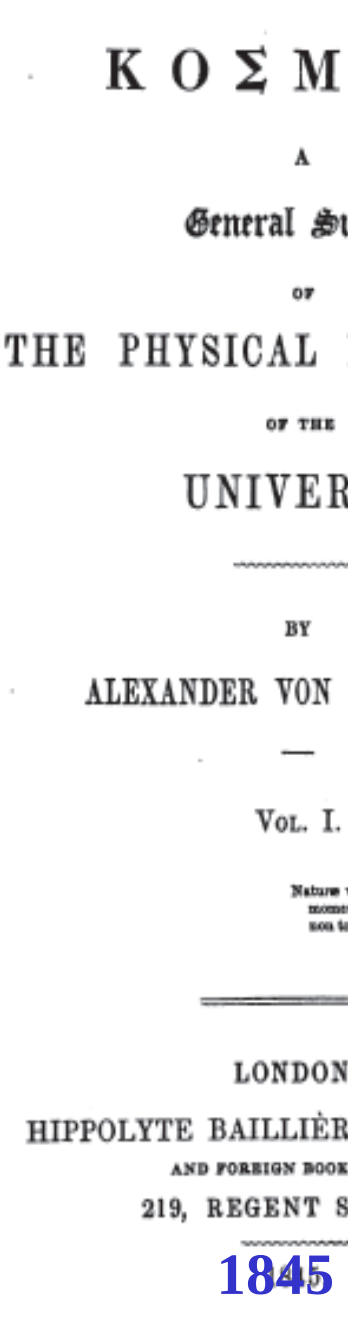
- **the founder of geography: national, physical and comparative; probably also environmental science;**
- **first uses graphic methods to show scientific results;**
- **makes the first isothermal map in the world.**

on systematic measurement with the most advanced instruments

the unity of nature → COSMOS in 5 vol

→ attempt to unify all sciences

www.deutschestextarchiv.de/book/show/humboldt_kosmos01_1845
www.humboldt-foundation.de



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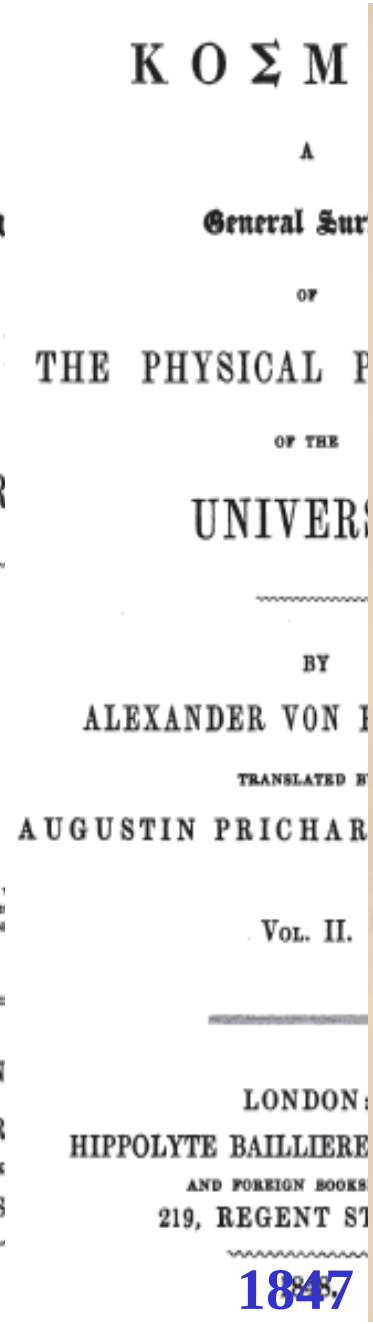

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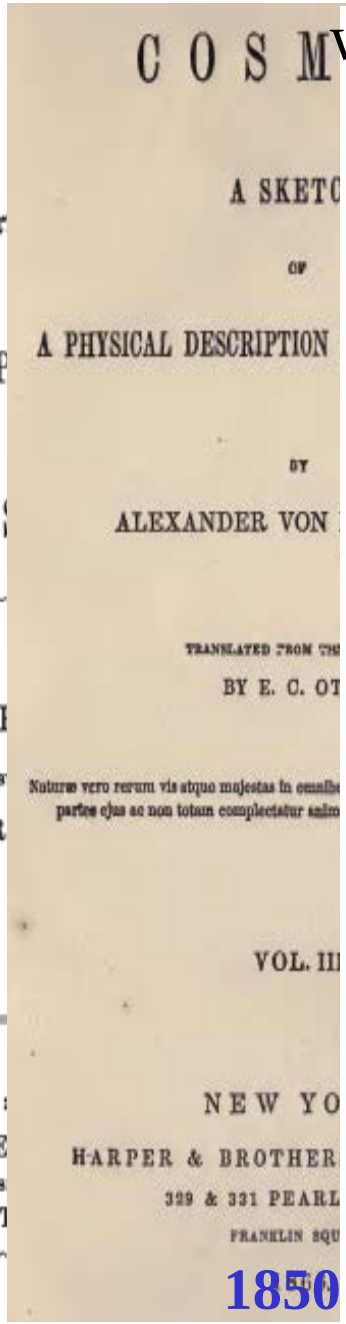

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OF

A PHYSICAL DESCRIPTION

BY

ALEXANDER VON

TRANSLATED FROM THE

BY E. C. OT

Naturæ vero rerum vis atque majestas in omnib

partes ejus ac non totam complectatur animo

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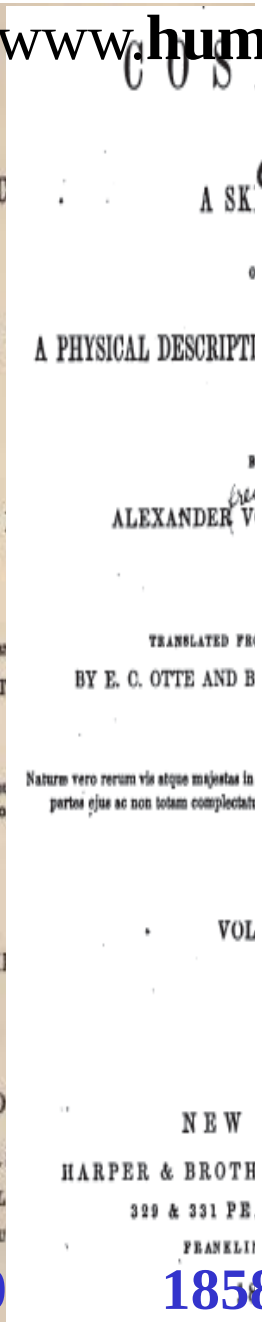

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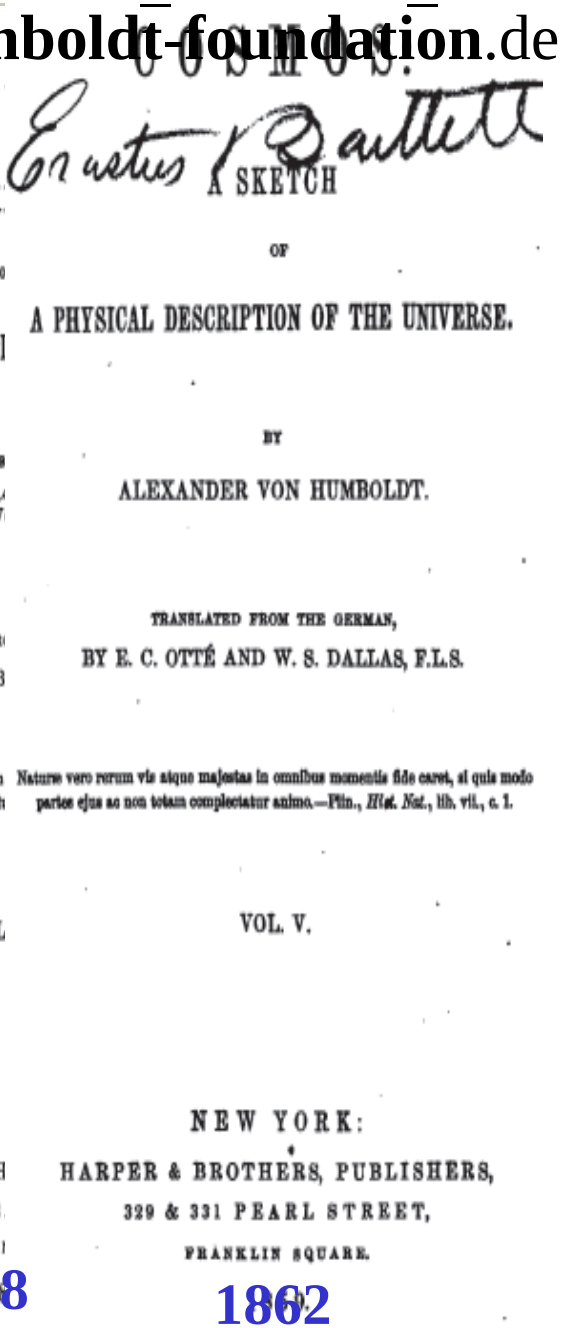


COSMOS.

Ernestus Dallett

A SKETCH

OF

A PHYSICAL DESCRIPTION OF THE UNIVERSE.

BY

ALEXANDER VON HUMBOLDT.

TRANSLATED FROM THE GERMAN,

BY E. C. OTTÉ AND W. S. DALLAS, F.L.S.

Naturæ vero rerum vis atque majestas in omnibus momentis fide caret, si quis modo partes ejus ac non totam complectatur animo.—Plin., *Hist. Nat.*, lib. vii., c. 1.

VOL. V.

NEW YORK:

HARPER & BROTHERS, PUBLISHERS,

329 & 331 PEARL STREET,

FRANKLIN SQUARE.

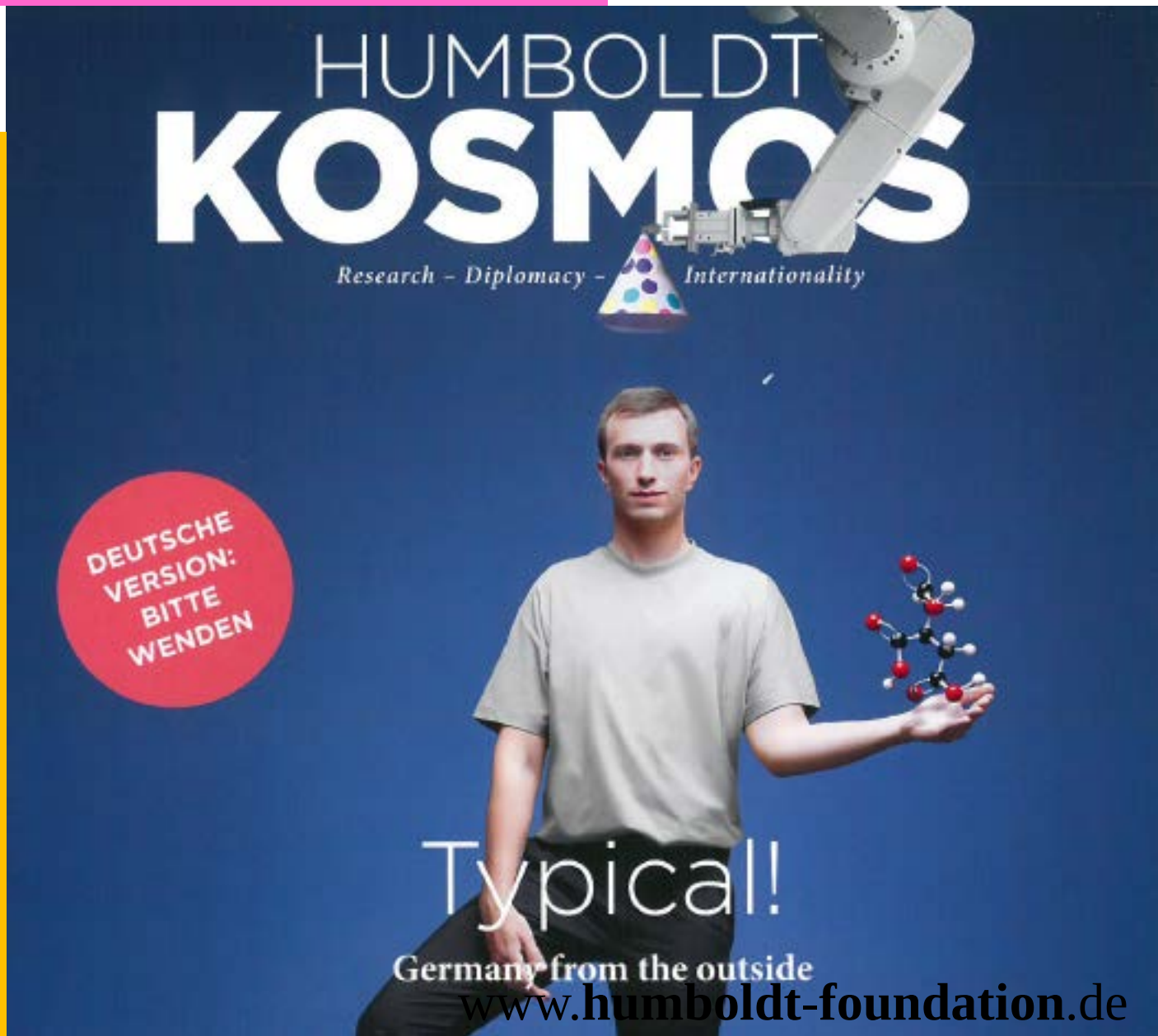
1862

Alexander von Humboldt Foundation Letters

Erik served Alexander von Humboldt Foundation for 18 years as a selection committee member.

Many of us came to Germany as a Humboldtian.

Thank you, Erik



II. The scientific work of Erik

As an old friend of decades, I am very pleased of being invited to attend this great celebration;

As an old friend of decades, I am gratefully honored to talk about Erik's scientific work at this celebration;

But I am still wondering whether I am able to talk about the scientific work by Erik, since Erik makes many great contributions to all aspects of geodesy.

A general impression on Erik's work:

- **Erik is great in all the aspects of geodesy;**
- **Erik's work is mathematically and physically fundamental to the geodetic science;**
- **Erik is the King of biaxial ellipsoids in the geodetic community worldwide. And of course,**

Each work from Erik requires a lot of time to study, understand and appreciate.

Erik's work up to the present:

18 books and 348 papers

covering all the aspects of geodesy and beyond

Erik's scientific papers up to the present

Liste der wissenschaftlichen V
Erik W. Grafarend

1. Der Rückwärtseinschnitt mit
surgskreisel
(zus. J. Kunz)
Bergbauwissenschaften *12* (19

2. Bergbaubedingte Deformatio
formationstensor
Bergbauwissenschaften *14* (19

3. Allgemeiner Fehlertensor bei
posteriori Korrelationen
Zeitschrift für Vermessungsw
157-165

4. Richtungsabhängiges tensoriel
pflanzungsgesetz
Allgemeine Vermessungsna
(1967) 300-337

5. Der klassische nord-süd-suche
Vermessungstechnische Rund
1-16, 33-39

6. Fehlertheoretische Untersuchu
nometrische Meßverfahren bei
Aufsatzkreiseln in Kombinat
elektrooptischen Entfernungsm
Geodimeter 4 B
Deutsche Geodätische Komm
Bayerischen Akademie der Wi
112, München 1967

7. Die Genauigkeit eines Punktes
Zeitschrift für Vermessungswe
107-111

8. Fehlertheoretisch günstigstes
schneiden
Zeitschrift für Vermessungswe
414-419

9. Nonlocal continuum mechanics
chi Geometry
Research Notes No. 141. Res
tion of Applied Geometry (R
1969

10. Chronometrische Nordbestimm
messungskreiseln
Zeitschrift für Vermessungswe
105-111

11. Die Methode der minimalen v
ten Varianz als mehrdimension
der Methode der minimalen Va
Zeitschrift für Vermessungswe
223-227

73. Schätzung von Varianz und Kovarianz der Beobachtungen in geodätischen Ausgleichsmodellen.
Allgemeine Vermessungsnachrichten *85* (1978) 41-49

Allgemeine Vermessungsnachrichten *78* (1971) 41-49

26. Mittlere Punktfehler und Vorwärtseinschneiden
Zeitschrift für Vermessungswesen *96* (1971) 41-54

27. Beobachtungsverfahren bei Vermessungskreiseln
Allgemeine Vermessungsnachrichten *78* (1971) 92-105

28. The free nonlinear boundary value problem of physical geodesy
Bull. Géodésique *101* (1971) 243-262

29. Isotropietests von Lotabweichungen Westdeutschlands
Z. Geophysik *37* (1971) 719-733

30. Lotabweichungsverteilungen Westdeutschlands und ihre gruppentheoretische Struktur
Mitt. Institut für Theoretische Geodäsie No. 1, Bonn 1971

31. The object of anholonomity and a generalized Riemannian geometry for geodesy
Boll. Geofisica teoretica et appl. *13* (1971) 241-253

32. A combined gravimetric-astrogeodetic method for tellaroid and vertical deflection analysis
Deutsche Geodätische Kommission bei der Bayerischen Akademie der Wissenschaften B 188, München 1971

33. Hilbert-Basen zur Optimierung mehrdimensionaler Punktmannigfaltigkeiten
Zeitschrift für Angewandte Mathematik und Mechanik *52* (1972) 240-241

34. Shar-Effekt-Tests an Vermessungskreiseln
Allgemeine Vermessungsnachrichten *79* (1972) 318-326, 420-421

35. Point-and interval-estimations, especially of point errors, in multidimensional least squares adjustment
Bulletin Géodésique *104* (1972) 165-184, 105 (1972) 319-342

36. Nichtlineare Prädiktion
Zeitschrift für Vermessungswesen *97* (1972) 245-255

37. Isotropietests von Lotabweichungsverteilungen Westdeutschlands II
Z. Geophysik *38* (1972) 243-255

Konzept stochastisc

38. Die freie geodätische Randwer
das Problem der Integrationsfl
der Integralgleichungsmethode.
Tagungsheft "Freie Randwertau
E. Grafarend /N. Weck. Mitt.
Theoretische Geodäsie No. 4,
Methoden und Verfahren der ma

49. Shar-Effekt-Tests
Kreiseltheodoliten
(zus. F. Halmos)
Allgemeine Vermessung
(1974) 176-179

50. Free adjustment of oscillatio
Allgemeine Vermessung
(1974) 170-176

51. A LOD and wobble analy
Euler-Liouville type based
equations
Proc. Symposium on Ear
Field and Secular Variation
es 643-659, Sydney/Australi

52. Optimization of geodetic ne
Bollettino di Geodesia e
(1974) 351-406

53. World geodetic datum WD
satellite and terrestrial obser
(zus. B. Eitschberger)
Bulletin Géodésique *114* (1

54. Eine Lotabweichungskarte
nach einem geodätisch ko
gorov-Wiener Modell
(zus. G. Offermanns)
Deutsche Geodätische Ko
Bayerischen Akademie der
82, München 1975

55. Cartan frames and a founc
geodesy
Methoden und Verfahren d
Physik, Bd. 12 (Eds. B. B
tensen), BI-Verlag, Mathe
Mannheim 1975

56. The geodetic boundary valu
Methoden und Verfahren d
Physik, Bd. 13 (Eds: B.Bro
tensen), BI-Verlag, Mathe
Mannheim 1975

57. Geodetic prediction concept
Methoden und Verfahren d
Physik, Bd. 13 (Eds. B. B
tensen), BI-Verl., Mather
Mannheim 1975

58. Geodetic stochastic process
Methoden und Verfahren d
Physik, Bd. 14 (Eds. B. B
tensen), BI-Verlag, Mathe
Mannheim 1975

59. Vermessungskreisel im System der dreidimensionalen Geodäsie

85. Kriterion-Matrizen I - zweidimensional homogene und isotr
Zeitschrift für Ve
133-149

86. Third-order desig
Int. Symposium
and Computation

in: Sneeuw et al (eds.), IAG Symposia 137 "VII Hotine-Marussi Symposium on Mathematical Geodesy", Rome, Italy, 6-10 June 2009, DOI 10.1007/978-3-642-22078-4_39, pp. 259-265, Springer Verlag Heidelberg, Berlin 2012

334. The MARUSSI Legacy: The Anholonomity Problem, Geodetic examples,
in: Sneeuw et al (eds.), IAG Symposia 137 "VII Hotine-Marussi Symposium on Mathematical Geodesy", Rome, Italy, 6-10 June 2009, DOI 10.1007/978-3-642-22078-4_2, pp. 5-15, Springer Verlag Heidelberg, Berlin 2012

335. Did a change in tectonic regime occur between the Phanerozoic and earlier epochs?
(zus. P. Varga, F. Krumm, C Doglioni, G Panza, F Riguzzi, A Schreider, N Sneeuw)
Rend. Fis. Acc. Lincei *23 (2012)* 139-148, DOI 10.1007/s12210-012-0172-6

336. Multivariate statistical analysis of deformation tensors: independent vs. correlated tensor observations
(zus. K. Moghtased-Azar, F. Tavakoli and H.R. Nankali)
Stud. Geophys. Geod. *56 (2012)* 977-992, DOI 10.1007/s11200-011-9024-6

337. Von A. Einstein über H. Weyl und E. Cartan zur Quanten-Gravitation
Sitzungsberichte der Leibniz-Sozietät der Wissenschaften *113 (2012)* 13-21, Berlin 2012

338. Evolution of the oceanic and continental crust during Neo-Proterozoic and Phanerozoic
(zus. P. Varga, F. Krumm, N. Sneeuw, A. Schreider, F. Horváth)
Rend. Fis. Acc. Lincei *25 (2014)* 255-263, DOI 10.1007/s12210-014-0298-9

339. Das duale wisschenschaftliche Paar Moritz – Molodenskij: Geodätische Höhen und Höhensysteme, in: *"Moritz80", 20 pages*, http://leibnizsozietaet.de/wp-content/uploads/2013/11/Grafarend_MoritzPaper_2014.pdf

340. The reference figure of the rotating Earth in geometry and gravity space and an attempt to generalize the celebrated Runge-Walsh approximation theorem for irregular surfaces
Int. Journal on Geomathematics *6 (2015)* 101-140, DOI 10.1007/s13137-014-0068-y

341. Fourth order Taylor-Kármán structured co-

Int. Journal on Geomathematics *6 (2015)* 319-342, DOI 10.1007/s13137-015-0071-y

342. Spacetime modeling of the Earth"s gravity field by ellipsoidal harmonics
(zus. M. Klapp and Z Martinec)
In: "Handbook of Geomathematics", eds. W. Freeden et al, 381-456, Springer Verlag, Berlin-Heidelberg 2015

343. Theory of Map Projections: From Riemann Manifolds to Riemann Manifolds
In: "Handbook of Geomathematics", eds. W. Freeden et al, 1-69, Springer Verlag, Berlin-Heidelberg 2014
DOI 10.1007/978-3-642-27793-1_53-1

344. The zero gravity curve and surface and radii for geostationary and geosynchronous satellite orbits
(zus. L.E. Sjöberg, M.S. Joud)
J. Geod. Sci. *7 (2017)*, 43-50, DOI 10.1515/jogs-2017-0005

345. Influence of Tidal Forces on the Triggering of Seismic Events
(zus. P. Varga)
Pure Appl. Geophys. *175 (2018)* 1649-1657, DOI 10.1007/s00024-017-1563-5

346. Relation of different type Love-Shida numbers determined with the use of time-varying incremental gravitational potential
(zus. P. Varga, J. Engels)
Pure Appl. Geophys., *175 (2018)* 1643-1648, DOI 10.1007/s00024-017-1532-z

347. The Global World of A. Dermanis and an attempt to use System Dynamics for the analysis of Polar-Motion (PDM) and Length of Day Variations (LOD), in: Quod Erat Demonstrandum – In quest of the ultimate geodetic insight, Special issue for Professor Emeritus Athanasios Dermanis, School of Rural and Surveying Engineering, p. 1-35, AUTH 2018

348. Anholonomity in Pre- and Relativistic Geodesy, in: "Relativistic Geodesy: Foundations and Applications", eds. D. Puetzfeld and C. Lämmerzahl, 228-289, Springer Nature Switzerland AG 2019, DOI 10.1007/978-3-030-11500-5

348 since the 1st paper in 1965

There are too many great publications from Erik for me to talk, and of course, likely beyond my capability.

I will have to briefly outline some of the contributions on the following topics:

- **map projection and geometry**
- **ellipsoidal representation and geodetic boundary value problem in physical geodesy and beyond**
- **linear and nonlinear statistical adjustment theory**
- **optimization and design of geodetic networks**
- **geodetic geophysics (likely from Prof. Peter Varga)**

great contributions to map projection and geometry

State of the art:

- **an old subject of study**
- **conformal, equidistant, equiareal ← not always consistent**
- **sphere or other manifolds**

Contributions from Erik

- ✓ **oblique Mercator projection: sphere → ellipsoid (JG 1995a)**
- ✓ **oblique azimuthal projection: → ellipsoid (JG 1995b)**
- ✓ **Mollweide projection: sphere → elliposid (JG 1995c)**
- ✓ **Hammer projection (azim trans rescaled equiareal): → ellipsoid (JG 1997)**
- ✓ **Korn/Lichtenstein equations (conformal): → ellipsoid (JG 1998)**
- ✓ **Harmonic mapping (min distortion energy): → ellipsoid (JG 2005)**

and more papers (too many to list here) on

systematic extension of map projections of ellipsoid

examples: differential geometry in geodesy

manuscripta geodaetica (1994) 19:339–345

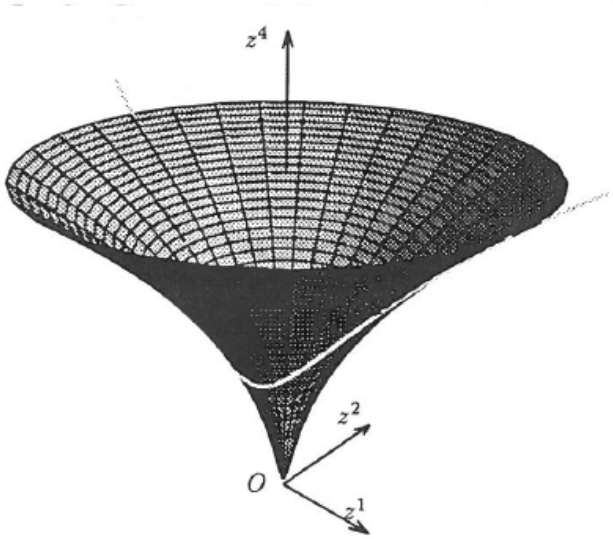
Newton mechanics as geo
the local isometric embed

H. Goenner [1], E. W. Grafarend [2], and F

[1] Institute for Theoretical Physics, University o
[2] Department of Geodetic Science, Stuttgart U

Received 12 July 1993; Accepted 6 October 199

Abstract According to the *Maupe*
of least action the Newton equations
mass point, namely in the gravitatio
can be interpreted as geodesic flow
tuis manifold. The *Maupertuis man*
formally flat threedimensional *Riem*
fold with the gravitational potential
of conformality. Here we give *the e*
immersion of different types of *Mau*
folds.



(b) Submanifold defined by eqn. 3(10) for $q_\phi = 0$ and a Kepler hyperbolic geodesic flow ($E > 0$)

Figure 1: Spherical symmetric Maupertuis manifold and Kepler geodesic flow

manuscripta geodaetica

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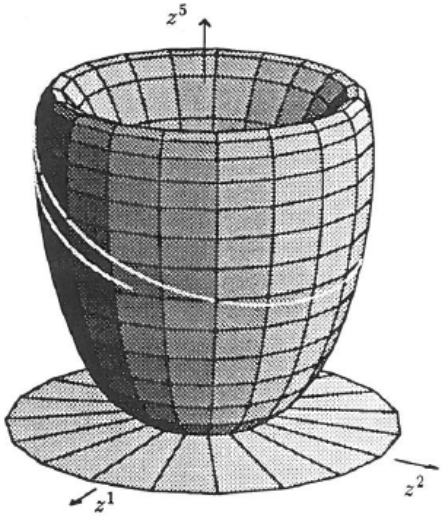


(b) Submanifold defined by eqn. 3(22) for $z^3 = 0, z^4 = 0, \Phi^2 := 2(E + \mu/q - \mu J_2 q_0^2 \cdot \cdot(3(q^3)^2/2q^5 - 1/2q^3))$ and a nonKepler geodesic flow ($E < 0$), where $J_2 = 1086.25 \times 10^{-6}$, q_0 : the earth's radius

Figure 2: Axialsymmetric Maupertuis manifold and geodesic flow

using the Maupertuis principle of least action to elegantly prove that the Newton equation of motion of a point mass in the gravitational field can be interpreted as a geodesic flow in a Maupertuis manifold.

examples: geometry of earth rotation

Int J Geomath (2011) 2:113–122
DOI 10.1007/s13137-011-0018-x

ORIGINAL PAPER

A minimal atlas for the rotation group $SO(3)$

Erik W. Grafarend · Wolfgang Kühnel

S1: Represent the earth rotation in quaternions, which is diffeomorphic to a 3D unit sphere, then

S2: Prove that the minimum number of maps of an atlas covering the entire rotation group SO(3) is equal to 4 and further

S3: Use the Euler or Cardan angles to construct such an atlas.

For more information, see Theorem on page 116.

great contributions to ellipsoidal representation and geodetic boundary value problem in physical geodesy

- solution to Stokes boundary value problem on an ellipsoid ➔ ellipsoid (Stud GG 1997)
- World Geodetic Datum 2000 ➔ for the best ellipsoid (JG 1999)
- ellipsoidal Bruns formula ➔ ellipsoid (JG 2001)
- National height datum, Gauss/Listing geoid level value (Baltic sea level project): ➔ ellipsoid (JG 2002)
- ellipsoidal vertical deflection and ellipsoidal gravity disturbance: case study ➔ ellipsoid (Stud GG2006)
- ellipsoidal spectral properties of gravitational potential and its first and second derivatives ➔ ellipsoid (JG 2005)
- ellipsoidal gravimetric/altimetry/astronomical boundary value problem: Iran case study ➔ ellipsoid (JGeody 2005)

and more papers (too many to list here) on

systematic extension of ellipsoidal representation

One example:

The Stokes BVP on an ellipsoid of revolution (SGG 1997)

$$\nabla^2 T = 0 \qquad \text{for } u > b_0 \ ,$$

$$\frac{\partial T}{\partial u} + \frac{2}{u} T = -f \qquad \text{for } u = b_0 \ ,$$

$$T = \frac{c}{u} + O\left(\frac{1}{u^3}\right) \qquad \text{for } u \to \infty \ ,$$

$$T(b_0, \Omega) = \frac{\alpha_{00}(u)}{4\pi} \int_{\Omega_0} f(\Omega')\, d\Omega' +$$

$$+ \int_{\Omega_0} f(\Omega') \sum_{j=2}^{\infty} \sum_{m=-j}^{j} \alpha_{jm}(b_0) Y^*_{jm}(\Omega') Y_{jm}(\Omega)\, d\Omega'$$

$$\alpha_{jm}(b_0) = -\frac{Q_{jm}\left(i\frac{b_0}{E}\right)}{\left.\frac{dQ_{jm}\left(i\frac{u}{E}\right)}{du}\right|_{u=b_0} + \frac{2}{b_0} Q_{jm}\left(i\frac{b_0}{E}\right)}$$

practical importance

relative error

spherical: 3.0x10^{-3}

ellpsoidal: 1.5x10^{-5}

absolute error

spherical 0.5m

ellipsoidal ➔ 2 mm

Erik is the King of ellipsoid in the geodetic community worldwide

for map projection and physical geodesy.

great contributions to linear and nonlinear statistical adjustment theory

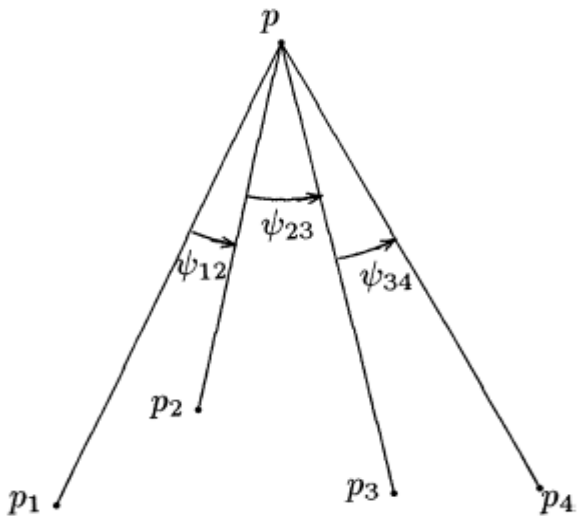
- close form solution to 3D resection

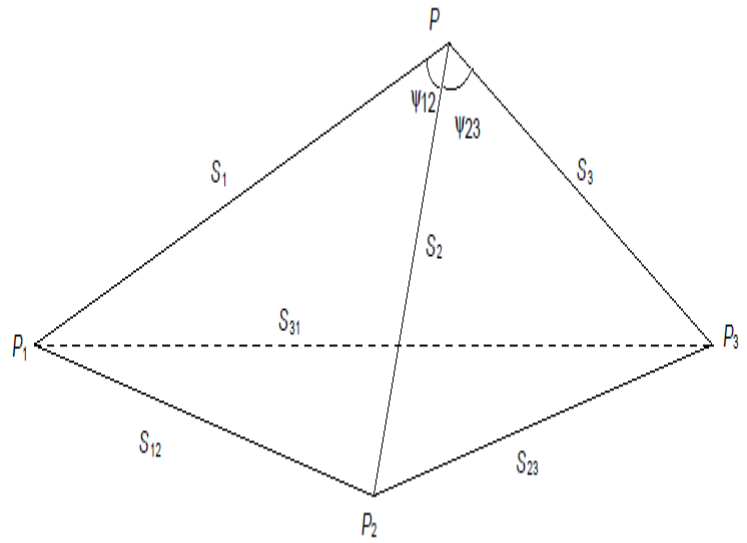
  ➔ 5 ZfV papers and many J Geod, EPS papers

  3-step method ➔ Moebius barycentric coordinates ➔ Groebner basis algorithm ➔ polynomial resultant approach

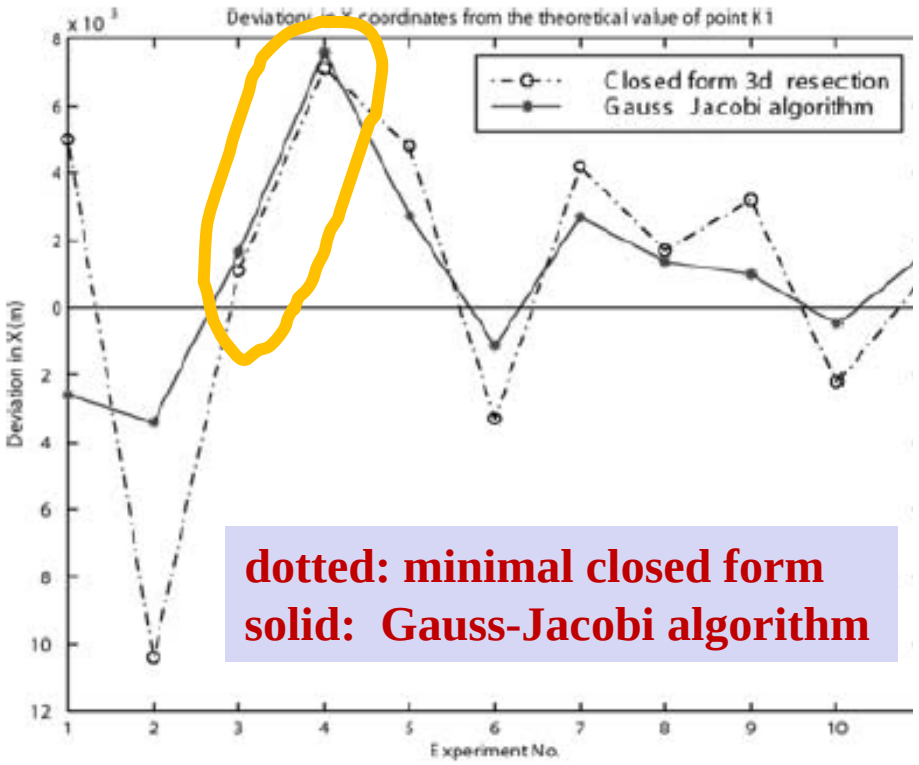
  minimal condition ➔ over-determined (Gauss-Jacobi combinatorial)

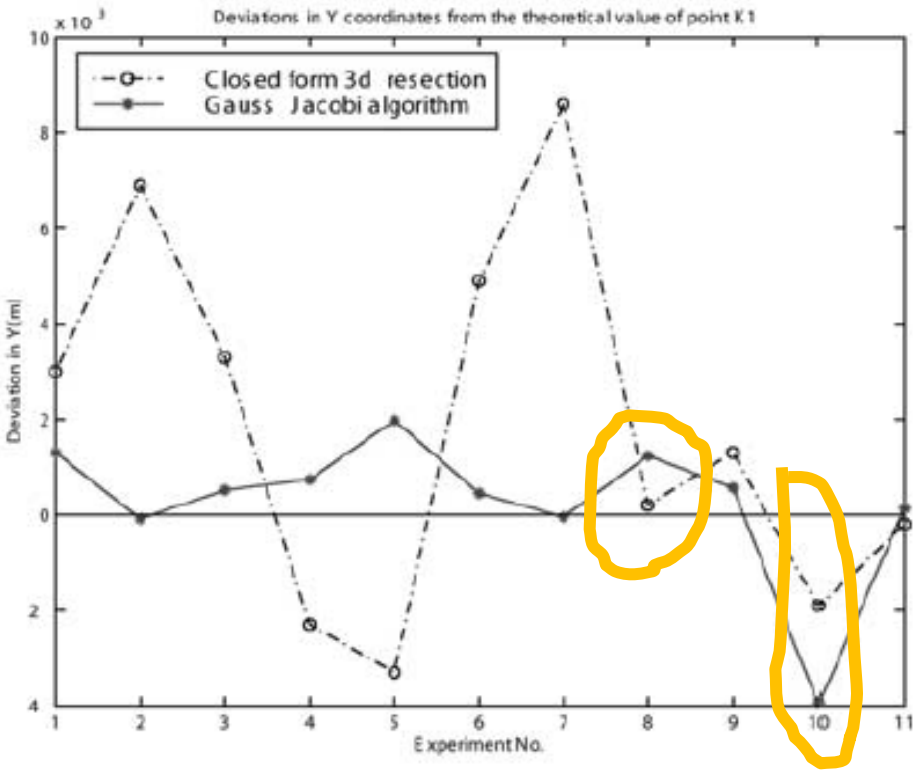
  angle measurements ➔ distance measurements





error analysis in the over-determined case





dotted: minimal closed form
solid: Gauss-Jacobi algorithm

Network: Stuttgart Central (2003, J Geod 76, 605-616)

➔

implying that a proper weighting scheme is required, since the errors with minimal condition are smaller.

- **extending the rank defect analysis to satellite network, and clearly deriving the rank deficiency for two types of satellite networks by assuming directional and distance measurements (papers on manuscripta geodaetica 1978)**

Type I: geometric and semi-dynamic mode, either with geometric Cartesian coordinates or Kepler elements

Type II: dynamic mode with gravitational force parameters

great contributions to optimization and design of geodetic networks

Erik pioneered the four classes of optimal design of geodetic networks

- ✓ **zeroth order design → datum definition**
- ✓ **first order design → configuration**
- ✓ **second order design → working scheme**
- ✓ **third order design → inserting new measurements**

This pioneering achievement, though likely not even the most proud work from Erik, has triggered a great number of research in geodesy for decades, in particular, during the 70s, 80s and 90s of the 20th century and has profoundly influenced, at least, one generation of geodesists. We can still find a lot of citations to Erik's work on this topic.

Optimization of Geodetic Networks

The Canadian Surveyor
1974, 28(5), 716-723

Erik W. Grafarend[1]

[1]Institut für Theoretische Geodâsie der Universitât Bonn 53 Bonn, W-Germany Nussallee 17

Published on the web 22 July 2019.

The Canadian Surveyor, 1974, 28(5): 716-723, https://doi.org/10.1139/tcs-1974-0120

ABSTRACT

Optimization of geodetic networks is classified in different orders. (1) Zero-order design solves the geodetic datum problem of how to transform geodetic measurements into absolute Cartesian coordinates. (2) First-order and (3) second-order design are closely related to find an optimal configuration and an optimal observational weight within a geodetic net. (4) As a third-order design, the problem of introducing new observations into an existing geodetic net referred to as geodetic Bayesian estimation is discussed. In the Introduction, the general Gauss-Markov model, BLUE, least squares and BLIMBE are reviewed. By experimental design, a geodetic experiment is defined. For geodetic loss functions of cost, time, material, on the one hand, and of the dispersion matrix of the net coordinates on the other, five different classes are introduced: invariants for point transformations, eigen values and condition numbers that have the property to produce a homogeneous and isotropic geodetic net, invariants of derived quantities, maximal distribution functions, and norms of the dispersion matrix. The interpretation of a geodetic net in terms of stochastic processes is reviewed, especially the Taylor-Karman decomposition (1) is solved by the Moore-Penrose inverse, the Tykhonov inverse and the bias problem. (2) gives a review of critical configuration designs, three examples for simulation and nonlinear programming of type A designs (scalar risk functions). Although the configuration design is strong nonlinear, many problems in observational weight design (3) are linear. For type B designs (given the explicit structure of the dispersion matrix of net coordinates) three-dimensional examples solved by simplex algorithms are presented. Literature on the Schreiber Theorem of optimal weight distribution in a geodetic net is collected. Finally in (4) the geodetic Bayesian approach is formulated.

III. Erik and Wuhan

- **Erik proactively promotes international collaboration by accepting many AvH Fellows and scientific visitors;**
- **A lot more hours of collaboration while I stayed in Stuttgart;**
- **A lot more hours of happiness and pleasure with Erik and Ulrike in their house, always a free concert by Erik and Ulrike;**
- **Deeply concerned with the future of the Institute of Navigation in Stuttgart, Erik took a decisive leadership to initiate the exchange between Stuttgart and Wuhan, together with Prof. J. Liu. I was pleased to be part of that process with Erik, which has academically led to a long lasting exchange of researchers and students.**

Honorary Professor of Wuhan University on Nov 3, 2000 Wuhan

I hereby cordially invite

Dr. Erik W. Grafarend

as an Honorary Professor of

Wuhan University.

Hou Jiecheng

President of
Wuhan University

敦 聘

格拉法伦特 博士

为 武 汉 大 学

名 誉 教 授

校 长

二〇〇〇 年十一月 三日

Concluding remarks

- **Erik is the greatest geodesist of the present. He is great in all the aspects of geodesy.**
  He makes fundamental scientific contributions to geodetic science, mathematically and physically;
- **Erik is the King of biaxial ellipsoids in geodesy for map projection and physical geodesy;**
- **Erik pioneers optimization and design of geodetic network .**
- **and far more beyond ….**

Finally,

I: again very happy birthday to you, Erik

II: and wish you always in your best health condition.

III: We are curiously waiting for reading your 1500 plus pages book to come.

happy birthday wishes and best regards to Erik on my way to Stuttgart from
Prof. Deren Li, Wuhan University
Prof. Karl-Rudolf Koch, Bonn University
Prof. Juergen Kusche, Bonn University
Prof. Harold Schuh, GFZ
Prof. Yunzhong Shen, Tongji University

Thank you very much!