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„Die Förderung der wissenschaftlichen Geodäsie
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Geodesy and Mathematics: Recent Developments in the Deep Rooted Relationship

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Introduction – historical beginnings

With its roots geodesy reaches back up to the peculiarities in the *philosophy of the ancient Greeks - to systematically collect information* on the *size and shape of the planets* and conditions on their surface, e.g. the distribution of waters and land, climatic conditions, habitability etc.

***To determine the shape and size of the Earth seemed to be beyond the power of man.* A simple observer with its senses does not even perceive the shape of the Earth. This seems to him to be a plain. What can be said about the simple observer, was true in the oldest time about all mankind in general.**

***Homer* (*900-800 B.C.*) expresses an opinion that the *Earth has a shape of a flat disc* which is surrounded by oceans. The presumption has for long time been regarded as correct, even though already at that time it contradicted the phenomenon in the appearance of a ship on the horizon that indicated the curvature of the surface of the sea level.**

Only *Pythagoras* (*580-500 B.C.*) and *Aristoteles* (*384-322 B.C.*) refuted the view of Homer and deduced that the ***Earth is spherical*. However, concerning the** *dimension* **of the globe we are missing any news.**

And only *in 220 B.C. Eratosthenes of Cyrene* (276-194 B.C.), Greek mathematician and astronomer, *made the first historically known experiment to determine the dimension* of the globe. From the height of the Sun and from the known distance between two places on the same meridian (*Alexandria-Syene*) he calculated that the diameter of the Earth is about *7150* km.

Geodetic science was born in France* and even in the beginning of the 19th century it was cultivated almost only by French. In its early days it was strongly influenced by the *large reversal in the development of physics*, which took place in the 17th century, during the period from *Galileo's experiments* to the formulation of the *fundamental laws of Newtonian mechanics.

The theory of universal attraction had an extraordinary charm for the philosophers of the Enlightenment period, **who used it as a weapon in the struggle against the remnants of feudalism.** *The question Newtonianism or Cartesianism was for some time a topic of great interest*, **not only in the circles of scholars, but also in the salons.**

A particular point of contention was the question of the shape of the Earth. **In the cosmogony defended by Cartesians the Earth was elongated at the poles. On the contrary from Newton's gravitational theory it results that on the poles the Earth is flattened.**

In the resulting dispute that divided the scientists into "*lemon shaped*" and "*flattened*" it was *geodesy and results of large degree measurements* **conducted by the French Academy of Sciences in the years** *1735-1737* **that played an active role.** *The results were a triumph of Newton's theory*.

Qualitatively ***higher demands on precision*** **of geodetic results that in the dispute should serve as arguments for an empirical verification of the consequences of Newton's theory,** ***gave rise to the development of the triangulation and degree measurements as a basis of geometrical methods for determining the shape and size of the Earth.***

It is definitely not surprising that geometrical methods could not permanently be enough for the determination of the shape and size of the Earth. When comparing various degree measurements ***differences appeared which could not be explained just by measurement and computational errors.***

Nevertheless, the ***finding of deflections of the vertical*** **has not stopped the fruitful development of the theory of the figure of the Earth.** ***It was only necessary to consider phenomena, which originally were omitted*** **in the theory, i.e.** ***gravitational effects*** **of the whole complicated inhomogeneous structure** ***of the Earth's body.***

Higher Geodesy in the Sixties

If now we move to the *60s of the past century*, we can see, that to a considerable measure *the solution of the problems above still has a dominant position among the scientific objectives of geodesy.* This was also reflected in *university curricula*.

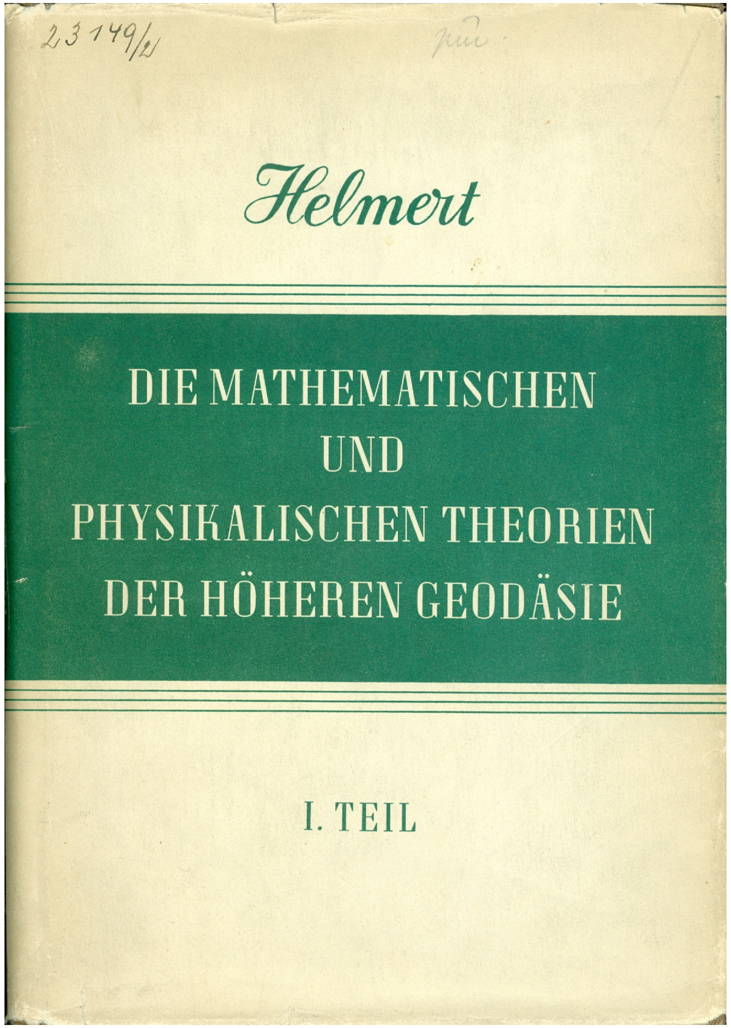
Textbooks contained *many references to Helmert's concepts* developed for the solution of problems in *Higher Geodesy*. The famous *two volumes* of *Helmert's book*

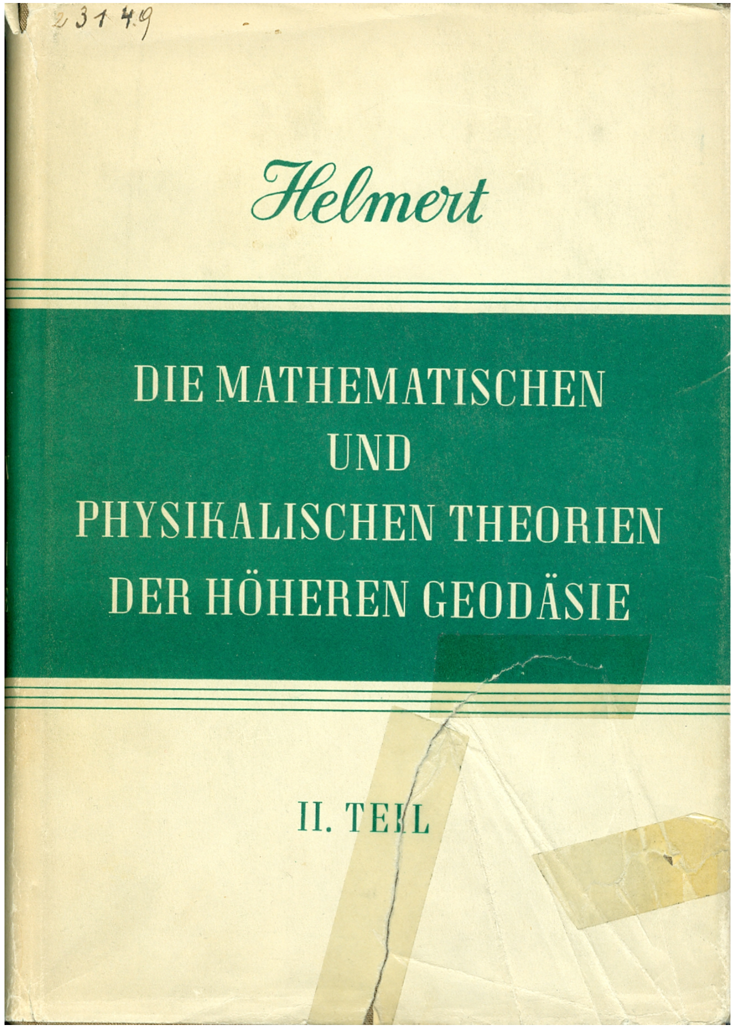
"Die mathematischen und physikalischen Theorien der höheren Geodäsie"

had an *explicit position* in the list of references and were *recommended for deeper studies*.

Recall that the *1st edition* of Helmert's book appeared in *1884* and was published by *B.G. Teubner in Leipzig.*

Later in *1962* the *2nd edition* was published by *B.G. Teubner Verlagsgesellschaft Leipzig*.





Mathematischen und Physikalischen Theorien der Höheren Geodäsie

Prof. Dr. F.R. Helmert

Einleitung

1. Gegenstand der Geodäsie.
2. Historische Entwicklung der Kenntnisse von der mathematischen Erdoberfläche.
3. Allgemeine mathematische Notizen, insbesondere Reihenentwicklungen.

I. Teil

Die mathematischen Theorien der höheren Geodäsie

1. Das abgeplattete Rotationsellipsoid.
2. Dreiecke und Dreiecknetze auf der Kugel.
3. Rechtwinklige und geographische Koordinaten auf der Kugel.

4. **Der vertikale Schnitt und das Sehnendreieck für das abgeplattete Rotationsellipsoid.**
5. **Fundamentalformeln für die geodätische Linie.**
6. **Differentialformeln und Reihenentwicklungen für die geodätische Linie.**
7. **Der Lauf der geodätischen Linie.**
8. **Das geodätische Dreieck: Dreiecksnetze.**
9. **Rechtwinklige geodätische Koordinaten und Übertragung geographischer Koordinaten mittelst derselben.**
10. **Berechnung kleiner Figuren auf dem Rotationsellipsoid mittelst Projektion auf eine Ebene.**
11. **Die Berechnungsarbeiten für eine Landesvermessung.**
12. **Messungen auf der physischen Erdoberfläche und näherungsweise Bestimmung einzelner Teil des Geoids.**
13. **Bestimmung des Erdellipsoids aus Gradmessungen.**

II. Teil

Die Physikalischen Theorien

1. **Allgemeine Eigenschaften der Niveauflächen.**
2. **Bestimmung der Abplattung aus Schweremessungen.**
3. **Ableitung einer Formel für die Schwerkraft im Meeresniveau aus den Beobachtungen; kontinentale Abweichungen des Geoids.**
4. **Synthetische Untersuchungen über die Einflüsse gegebener Massen auf die Niveauflächen in der Nähe der Erdoberfläche.**
5. **Zeitliche Änderungen der Niveauflächen.**
6. **Verwertung astronomischer Angaben für die Erkenntnis der Erdgestalt und des Erdinnern.**
7. **Das geometrische Nivellement.**
8. **Die trigonometrische Höhenmessung.
Mit Bemerkungen über die Lateralrefraktion und die Aberration.**

Going through the books, an idea definitely appears *what are the mathematical topics and tools applied* in the solution of the problems that are discussed?

Yes, the applications are fairly specific. In the *1st part* of the book they are mostly *related to the surface of an oblate ellipsoid of revolution and its close neighborhood.*

Nevertheless, concerning the 1st part we have in particular to mention:

- **Theory of Series**
- **Spherical Trigonometry**
- **Differential and Integral Calculus**
- **Differential Geometry of Curves and Surfaces**
- **Theory of Ordinary Differential Equations**

The concepts and results here represent what is a *kind of classic in geodesy.*

The nature of *the 2nd part* is different, it *is physical* and concerns the *body of the Earth*, its *gravity field* and also the *dynamics of its rotation*.

Here we can meet with problems governed by:

- **Ordinary Differential Equations**
- **Systems of Ordinary Differential Equations**
- **Partial Differential Equations**

It is this part of geodesy which was *most strongly influenced by an unprecedented period of accelerated development at the turn of 50s and 60s of the last century. Artificial satellites of the Earth came into play:*

- **The first was launched on *October 4, 1957.***
- **Soon the first *computation of the flattening of the Earth based on the observation of the motion of the satellite No. 1957β* appeared in *Nature* , Vol. 182, No. 4629 (*July 19, 1958*).**

The Cosmic Age has Changed the Situation

Geodesy appeared in a new light. ***The mathematical and physical foundations of geodesy were visibly demonstrated:***

- **strong impulses accelerated the development of geodetic methods based on geometrical principles,**

but geodesy also strengthened its position as a field for very fruitful application of

- **Celestial Mechanics**
- **Potential Theory**
- **and the Theory of Equations of Mathematical Physics in general.**

At universities the study of geodesy became more demanding, but also very attractive. The situation was not different at the international level.

It is very instructive to go through the development of theory oriented structures of, e.g., IAG

IAG Section IV – Theory and Evaluation

Period 1979-1983 **President:** L.P. Pellinen (USSR)

Secretaries: E.W. Grafarend (Germany), F. Halmos (Hungary)

SSG 4.56 - E.W. Grafarend (Germany)
Differential Geometry of the Grav-ity Field

Sub-Group - C. Boucher (France)
Relativistic Aspects of Differential Geodesy

SSG 4.57 - F. Sansò (Italy)
Boundary Value and Convergence Problems in Physical Geodesy

Sub-Group - P. Holota (Czechoslovakia)
Improperly Posed Problems

SSG 4.58 - H.M. Dufour (France)
Representation of the Gravity Field

SSG 4.60 - K.R. Koch (Germany)
Statistical Methods for Estimation and Testing of Geodetic Data

SSG 4.65 - E. Tengström (Sweden)
Force Function of Two or More General Bodies: Application for Geodynamics

SSG 4.66 - C.C. Tscherming (Denmark)
Management of Geodetic Data

SSG 4.70 - K.P. Schwarz (Canada)
Gravity Field Approximation Techniques

SSG 4.71 - G. Schmitt (Germany)
Optimization of Geodetic Networks

IAG Section IV – General Theory and Methodology

Period 1983-1987 **President:** E.W. Grafarend (Germany)

Secretaries: K.-P. Schwarz (Canada), F. Sansò (Italy)

SSG 4.56 - E. Livieratos (Greece)
Differential Geometry of the Gravity Field

SSG 4.57 - P. Holota (Czechoslovakia)
Boundary Value and Convergence Problems in Physical Geodesy

SSG 4.60 - D. Fritsch (Germany)
Statistical Methods for Estimation and Testing of Geodetic Data

SSG 4.66 - A.U. Frank (USA)
Geodetic Data Base Management

SSG 4.71 - G. Schmitt (Germany)
Optimal Design Problems

SSG 4.91 - H. Sünkel (Austria)
Local Gravity Field Approximation

SSG 4.92 - L. Sjöberg (Sweden)
Global Gravity Field Approximation

SSG 4.93 - P. Forsyth (Canada)
Wave Propagation in Refractive Media

SSG 4.94 - J. Wahr (USA)
Theory of Geodetic Reference Frames

SSG 4.95 - K.H. Ilk (Germany)
Multi Force Function: Geodetic Aspects of Astrodynamics

SSG 4.96 - P. Vaníček (Canada)
Models for Time-Dependent Geodetic Positioning

IAG Section IV – General Theory and Methodology

Period 1987-1991 **President:** K.-P. Schwarz (Canada)

Secretaries: F. Sansò (Italy), P. Holota (Czechoslovakia)

SSG 4.91 - R. Forsberg (Denmark)

Local Gravity Field Approximation

SSG 4.92 - H.-G. Wenzel (Germany)

Global Gravity Field Approximation

SSG 4.93 - F.K. Brunner (Austria)

Wave Propagation in Refractive Media

SSG 4.115 - F. Sacerdote (Italy)

Mathematical Analysis of Geodetic Boundary Value Problems

SSG 4.116 - K.-P. Schwarz (Canada)

Kinematic and Dynamic System Modelling in Geodesy

SSG 4.117 - D. Delikaraoglu (Canada)

Optimization of Modern Positioning Techniques

SSG 4.118 - K.H. Ilk (Germany)

Inverse Geodetic Problems

SSG 4.119 - E.W. Grafarend (Germany)

Relativistic Effects in Geodesy

SSG 4.120 - P.J.G. Teunissen (The Netherlands)

Non-Linear Adjustment

IAG Section IV – General Theory and Methodology

Period 1991-1995 **President:** F. Sansò (Italy)

Secretaries: P. Holota (Czech Republic), P.J.G. Teunissen (The Netherlands)

Special Commission SC1 - President: E.W. Grafarend (Germany)

Mathematical and Physical Foundations of Geodesy

Subcommission 1 - B. Schaffrin (USA)
Statistics

Subcommission 2 - R. Klees (Germany)
Numerical and Approximation Methods

Subcommission 3 - F. Sacerdote (Italy)
Boundary Value Problems

Subcommission 4 - J. Zund (USA)
Differential Geometry

Subcommission 5 - A. Drozyner (Poland)
Theory of Orbits and Dynamics of Systems

SSG 4.138 - A. Kleusberg (Canada)
Modelling and Quality Control for Precise Integrated Navigation

SSG 4.139 - A. Geiger (Switzerland)
The Role of Terrain in Gravity Field Modelling

SSG 4.140 - T. Spoelstra (The Netherlands)
Tomography of the Atmosphere by Geodetic Measurements

SSG 4.141 - R. Barzaghi (Italy)
Integrated Inverse Gravity Modelling

SSG 4.142 - B. Heck (Germany)
Application of the Boundary Value Problem Techniques to Space and Airborne Gravity Field Observations

IAG Section IV – General Theory and Methodology

Period 1996-1999 **President:** P. Holota (Czech Republic)

Secretaries: B. Heck (Germany), C. Jekeli (USA)

Special Commission SC1 - President: E.W. Grafarend (Germany)

Mathematical and Physical Foundations of Geodesy

Subcommission 1 - A. Dermanis (Greece)

Statistics

Subcommission 2 - W. Freeden (Germany)

Numerical and Approximation Methods

Working Group - R. Klees (The Netherlands)

Comparison of several techniques for solving geodetic boundary value problems by means of numerical experiments

Subcommission 3 - E.W. Grafarend (Germany)

Boundary Value Problems

Subcommission 4 - J. Zund (USA)

Geometry, Relativity, Cartography

Subcommission 5 - R.J. You (Taiwan)

Theory of Orbits and Dynamics of Systems

SSG 4.168 - P. Knudsen (Denmark)

Inversion of Altimetric Data

SSG 4.169 - B. Benciolini (Italy)

Wavelets in Geodesy

SSG 4.170 - L. Ballani (Germany)

Integrated Inverse Gravity Modelling

SSG 4.171 - L.E. Sjöberg (Sweden)

Dynamic Isostasy

SSG 4.176 - D. Wolf (Germany)

Temporal Variations of the Gravity Field

We can clearly see that *geodesy is indeed interdisciplinary* here. The *spectrum of mathematical topics involved is fairly broad.* In particular it concerns applications of:

- **Mathematical Analysis, Algebra, Theory of Linear Spaces**
- **Theory of Function Spaces and Function Analysis**
- **Boundary Value Boundary Value Problems for Ordinary Differential Equations and Systems of Ordinary Differential Equations**
- **Boundary Value Problems for Partial Differential Equations**
- **Improperly Posed Problems**
- **Numerical and Approximation Methods**
- **Differential Geometry, Tensor and also Cartan Calculus**
- **Relativistic Problems**
- **Statistics and Estimation Theory**
- **Optimization Problems, Adjustment Theory**
- **Management of Data, etc.**

It is also worth mentioning that the *way of reasoning* applied had a real or typical mathematical nature, i.e.:

assumptions - proposition - proof

Some results were indeed exceptional and of real mathematical nature. This definitely concerns *Bomford Prize Awards* of the International Association of Geodesy in 1975, 1979 and 1987 (E.W. Grafarend, F. Sansò, P.J.G. Teunissen).

The famous *Hotine-Marusi Symposia became a regular platform* for presenting mathematically oriented research in geodesy.

***Contacts with mathematicians were developed*. Solutions of some key problems in geodesy were considerably enriched.**

This concerns e.g. the involvement of *Lars Hörmander (1931-2012, Lund University)* who - as is known - was awarded the Fields Medal in 1962, the Wolf Prize in 1988 and Leroy P. Steele Prize in 2006.

Much was done by geodesists in leading positions of the IAG. Especially of those who had close tie and understanding for mathematics and physics.

Note also that within this development the original title

Theory and Evaluation

of IAG section IV was changed to

General Theory and Methodology

in 1983.

For sure it is important to make an analysis of the nature of geodesy in the new era.

In this connection, it is maybe useful to *recall an interesting material compiled in the beginning of this millennium:*

The IAG-Review 2000-2001 - Executive Summary

was circulated within the geodetic communuity.

In the introduction a *reference to Webster's dictionary* was used as a *"springboard"* for discussion.

***Geodesy is*, according to Webster's dictionary,**
"the branch of applied mathematics concerned with the determination of the size and shape of the Earth, and the exact position of points on its surface, and with a description of variations in its gravity field".

***In the background of the discussion* there was also an idea ventilated at the *plenary session in Birmingham, 1999*.**

I am quoting:
"geodesy may be viewed as an "auxiliary science" (very much like mathematics from the point of view of a physicist) by other scientists". The same idea then appeared in the Proceedings.

We can consider an aspect like this.

As an example let us quote what illustrates a certain feature in the *attitude of the famous Wolfgang Pauli to Pascual Jordan:*

"Herr Jordan was always a formalist",

he once said, meaning that *Jordan was not a true physicist but only a mathematician* - a lower form of life.

Note, however, that of the *triumvirate* (with *Born* and *Heisenberg*) that formulated *quantum mechanics* in the

***famous Dreimännerarbeit* of 1925,**

Jordan* was the *principal architect of the theory

Maybe this illustration will have an effect contributing to the self-reliance of geodesists.

In science it is perhaps not necessary to use a *superior-inferior classification* when considering individual disciplines. But when reading in the Executive Summary that:

> ***"Geodesy should in the future be seen as much more than applied mathematics and an observing system . . ."***

it somewhat resembles me the aforementioned attitude attributed to physicists.

Nevertheless, we can agree with the IAG Executive Summary:

***Geodesy today and in the future requires* a broad spectrum of *synergetic activities*, including theory, science, engineering, technology development, observations, analysis, and the development of practical-oriented services, making access to data and methodology easy and transparent across the whole spectrum of the geodetic community:**

i.e. national and regional survey authorities, survey and engineering companies, research institutions, and space agencies.

But the thing is somewhat sensitive* and *when we read that the task of IAG is to show that geodesy is an integral part of geo-information industry* (see the inauguration speech of the IAG president in 2011), *we definitely pay attention to the claim like this.

The discussion on geodesy is very instructive and in case it involves also mathematics *we can find some similarities* that are ventilated by *Philips A. Griffiths* in his paper titled *"Mathematics at the Turn of the Millennium"*.

The paper was presented *at the Conference "Frontiers of the Mind in the 21st Century" in the Congress library in Washington, 1999*. As professor of mathematics Griffiths was active at Duke University, in Harvard, Princeton and Berkeley

Among others *Griffiths* writes: *"The globalization, interactivity, and "opening out" of the mathematical enterprise is new and powerful trend."*

In this connection we definitely are motivated to remember the famous German mathematician

Felix Klein (1849-1925) **and his view of geodesy:**

***„Die Geodäsie ist im allgemeinen ein glänzendes Beispiel dafür, was man mit der Mathematik in den Anwendungen machen kann und wie man es machen soll*. Man bekommt selbstverständlich alles nur approximativ bestimmt, zugleich aber hat man überall da, wo die Untersuchung als zu Ende geführt gilt, das Mass der Annäherung festgestellt.“**

— — —

***At the very end we perhaps can mention still other aspects*. For instance in differential equations and mathematical modelling we can see a certain *metaphor for the real world*. Thus with our “mathematical world” we are close to art, but this is another large and inspiring chapter.**

Thank you
for your attention !