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Processing of Information in Microstates

Results of an elementary analysis of human information-processing, reconsidered[1,2]

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Summary: In Part 1 of this contribution it is shown that information can be measured by the synchronicity between EEG signals. In Part 2 it is shown that the multimodality hypothesis is confirmed by a topological consideration. In Part 3, the processing of information by humans is represented as a Markov chain of microstates which are based upon time-independent synchronicity between EEG signals. Experimental evidence is adduced supporting the assertion that mental performance is expressed in the degree of concatenation of microstates, quantifiable as a decrease in entropy.

Part 1: Information

In Jena, 27 years ago, there began a highly constructive collaboration between the mathematician Bärbel Schack, the mathematics didactician Frank Heinrich, the mathematician Gert Grießbach, the medical scientist Mathias Rother and the physicist Erdmute Sommerfeld. At various later points they were joined by diploma and doctoral students as well as post-doctoral and advanced research workers in psychology: Uwe Kotkamp, Gundula Seidel, Martin Grunwald, Heiko Tietze, Jürgen Ptucha, Henning Gibbons, Bettina Kriese, Christine Schleußner, Thomas Hübner, Ralf Goertz, Nicole Kotkamp, Sven Oelsner und Mauricio Parra. There also arose long-distance partnerships with Dietrich Lehmann (Zürich), Jürgen Bredenkamp (Bonn), Dieter Heller (Aachen), Dietrich Dörner (Bamberg), Rudolf Groner (Bern) and Gerd Lüer (Göttingen). We were all motivated by the question of whether the measure for cognitive effort associated with the differentially sensitive retention of information found at the model level can be undergirded by EEG measurements. At that time, nobody could have suspected that this would lead to the proposal that entropy decrease can be a measure of mental performance. Today, twelve years after the cessation of this work, we have attained a sufficient distance from it to reconsider it from a new perspective, to separate its more from its less important aspects, to point up the gaps and to position the problem more accurately. For this reason, we shall not be concerned with the methodological details here. This article is written in the (not entirely unselfish) hope that interested research groups will want to repeat these experiments, in order to obtain further support for – or indeed to falsify – the assertion that entropy decrease is a measure of mental achievement.

1.1 Objective

A new measurement criterion for cognitive processes presupposes a knowledge of what cognitive processes are. Discrete processes can be described as sequences of states and sometimes they can be quantified as Markov chains. This requires the states to be specified.

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1 In memory of Bärbel Schack. Her tragically early death tore her out of the middle of an eventful and creative period. Without her innovative mathematical work on adaptive EEG coherence analysis, the results on entropy reduction as a measure of mental performance would not have been attained.

2 I thank Peter Petzold, Erdmute Sommerfeld and Gerd Lüer for their critical reading of an early version of this text. The text was translated into English by Paul Woolley, whom I also thank for valuable suggestions. The German version: www.leibnizsozietaet.de/wp-content/uploads/2015/12/WKrause.pdf

For the special case of elementary problem-solving, Markov chains have been applied to the analysis of changes in strategy, whereby the states were strategies (Krause, 1970). The rules for a change in strategy in elementary problem-solving have been specified in this way. Groner (1978) showed in his book *Hypothesen im Denkprozess* (Hypotheses in the Thought Process) that the acquisition of strategies in problem-solving can also be described by Markov chains. Nevertheless, these approaches remain restricted to the specially selected requirements of the situation, as the states of the Markov chains have to be defined specifically, in each case, to suit these requirements. Therefore, a general approach to process analysis by Markov chains has so far not been possible.

For arbitrary requirements, this process analysis by Markov chains can be applied if the states for the requirements are determinable. This is precisely what is to be shown, insofar as EEG coherence microstates (Lehmann, 1987) are considered as states of the Markov chains. From the linkage between the states, measured by conditional probabilities, orders of the Markov chain can be calculated for arbitrary requirements and are then placed in a relationship to mental performance. This is proposed as a measure of mental performance and is taken up in Part 3.

One can pursue the claim to generality still further. If one regards the EEG coherence microstates as “letters of an alphabet” and assigns a meaning to each of them, then the microstate sequences would be given by “words” that would in some way represent the cognitive process. Thus, one must ask what the rules are that generate such sequences, rules of a “mental grammar”. However, at present we cannot go down this path, as the assignments of microstates to their respective meanings are as yet unknown. We have to restrict ourselves to the syntactic properties of a grammar.

The standpoint of association psychology, with the view that thinking is a concatenated sequence of mental images, does not immediately provide any explanation. However, the idea of a sequence (whatever it is a sequence of) is clearly expressed. In the course of the development of these notions, a classification of four levels has emerged, in which mental processes are pictured as sequences:

– The level of action, i.e., the level of external representation,
– The cognitive structural level, the level of internal representation,
– The microstate level, the level of neuroscience,
– The molecular-biological level, the level of epigenetics.

For the molecular-biological level, a separate treatment by experts in this field is needed. We refer here to the work of Müller (2010) or Spork (2012).

Formally, such sequences can be represented as transformations of external states Z_i, Z_j, of cognitive structures S_i, S_j and of microstates M_i, M_j by the operators Q^A, Q^S and Q^M:

– The action level: Q^A: $Z_i \rightarrow Z_j$
– The cognitive-structural level: Q^S: $S_i \rightarrow S_j$
– The microstate level: Q^M: $M_i \rightarrow M_j$

The advantage of the assumption of sequences consists in the fact that the properties of a process can be illustrated in a natural manner in the properties of sequences – that is, in the nature of the concatenation of elements. (In science this sequential consideration is frequently applied, because it requires very few assumptions.) The difficulty consists in determining the elements and their properties, and the challenge is to determine the interactions between the levels, especially as the time-scales of events at the various levels are very different. At the action level we are operating on a scale of minutes, at the cognitive-structural level in the region of seconds, at the microstate level in milliseconds and at the molecular-biological level the entire span of time-scales must be considered.

The topic of sequences at the action level and their relation to the cognitive-structural level was recently delineated by Heinz-Jürgen Rothe (2012) in his concluding lecture at Potsdam University. A consideration of judgement sequences has been published by Peter Petzold and Gerd Haubensack (2001, 2004). The cognitive-structural level, its modelling and its measurement have been set out by Erdmute Sommerfeld (1986, 1991, 1991a, 1994) and by Erdmute Sommerfeld and Werner Krause (2013). I shall restrict my considerations here to the level of microstates.

We set out from the general idea of a sequence of elements. For the process analysis at all levels we then have the following situation: We seek an operator Q, which transforms the element E_i into the element E_j.

At the action level, sequences of action in problem space are represented by the transformations of states: We are seeking an operator Q^A, which transforms a state Z_i (such as a particular constellation of discs in the "Towers of Hanoi" game) into a state Z_j such that – for example – the distance to the aimed-at state is reduced.

If a sequence of thought is to be represented in a sequence of actions, then properties of the thought sequence must be reflected in properties of the action sequence, for example in the degree of order of a sequence; in this way they must be expressible in numbers. It is presumably due to the great complexity and diversity of units at the action level that this approach has hitherto been unsuccessful. There is disagreement as to how a unit (or here, in the usual terminology, a state in problem space) is to be defined.

For the cognitive structural level a similar statement applies: We are seeking an operator Q^s, which transforms a cognitive structure S_i into a cognitive structure S_j – that is, which captures the change from one internal representation to another. At present it is possible to demonstrate experimentally for a given class of problems the existence of a fully-formed cognitive structure in dependence upon a given external representation. Measurement techniques have yet to be developed for studying how a cognitive structure S_j is generated from an existing cognitive structure S_i in a sequence – that is, for a structure sequence. The analysis of sequence properties to measure mental performance at the cognitive level is therefore (so far) unachievable. On the other hand, it has been shown, for the formation of cognitive structures in dependence upon a given external representation, that the property of reduction of effort to maintain this structure (in the special case, the number of attributes needed to retain and to reconstruct a cognitive structure) could well be used as a measure of mental performance (Krause, Sommerfeld, Höhne und Sperlich, 1998; Krause, 1991; Sommerfeld, 1994).

Finally, we can also formulate for the neurological level: We are seeking an operator Q^M, which transforms a microstate M_i (a particular electrical activity in the neural network) into another microstate M_j. It is that process that we shall deal with in the rest of this article.

First, we wish to consider the measurement variable for quantifying the elements in the sequence and to answer the question of whether EEG coherence really measures information (Part 1). We then examine the elements themselves, that is, their microstates and their properties (Part 2). Our concluding task will be to show that with this kind of process analysis, at this neurological level, the concatenation properties allow us to identify not only diagnostic metrics, but also active principles such as the simplicity principle (Part 3).

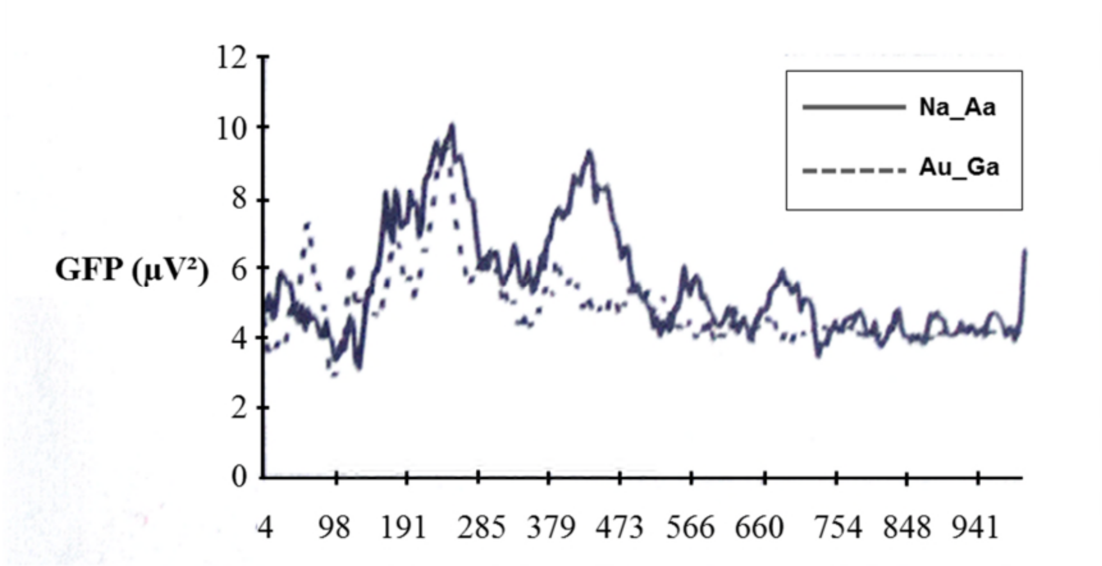
1.2 Measurement of information

The following considerations refer to EEG measurements. Traditionally, EEG analysis is based upon the amplitude of the EEG signal. However, this tacitly assumes that the information-processing steps in the brain are represented by the amplitude of, and the amplitude changes in, the EEG signal. In this way information is represented by a measure of performance, if one considers the square of the amplitude. Strictly speaking, however, information should be measured on an information scale and performance on a performance scale. How can this be resolved?

1.2.1 Performance

A conventional evaluation of EEG results – as can be done, for example, by determining the "global field power" (GFP; Lehmann, 1987) as a spatial standard deviation of all amplitude values at every time point across all EEG channels – indeed shows differences between two chosen sets of conditions (which we denote respectively as concept activation and dot-pattern comparison). However, it fails to allow any further, detailed investigation of the origins of these differences. In concept activation, for example, the subject is required to decide whether two letters “A” and “a” have the same name (abbreviated: Na_Aa); in dot-pattern comparison the subject must decide whether two dot patterns, as designed by Garner (1970), look similar (abbreviated: Au_Ga). Concept activation is in this case unnecessary. These classical experiments on comparison processes go back to the work of Posner and Mitchell (1967). Here we have chosen two cases out of a set of variations in conditions (Krause *et al*., 1997; 1998). Figure 1 shows the course of the global field power for the two sets of conditions

GFP concept / pattern



Time (ms)

Figure 1. Global field power (GFP) for the conditions Na_Aa (concept activation) and Au_Ga (dot-pattern comparison). Taken from the diploma thesis of Heiko Tietze (1996); see also W. Krause (1997); reproduced by kind permission of the publisher)

“concept activation, Na_Aa” and “dot-pattern comparison, Au_Ga”. (The methodological details are described by Krause (1997).) Both curves both show a peak at 250 ms. For “concept activation Na_Aa” a second peak appears at 420 ms, and one would be inclined to speak of a sensitive measurement variable. It would, however, be pure speculation to assign the common peak at 250 ms to a common information-processing operation and the peak at 420 ms to concept activation. Nevertheless, the results of numerous experiments support the sensitivity of the performance measure for cognitive processes. Here, an explanation is required, and we shall return to this at the end of the next point.

1.2.2 Synchronicity

By using the concept of “global field power” we can project the information-processing operation onto a scale of performance. Although this is usual in der psychophysiological practice, it is not strictly correct, as performance must be measured on a scale of performance and information on a scale of information. In our own analytical studies we have used EEG c*oherence* as the metric of choice. The coherence function is a measure of the linear relationship between two EEG-signals x(t) and y(t), derived via the frequency. Schack (1999) has shown the relationship between coherence and negative entropy, that is, between the selected metric and information; we reproduce this briefly here (the full argument can be found in the paper by Krause (2000)).

For two EEG-signals x and y, S_{xx} and S_{yy} are the autospectra of the individual signals and S_{xy} – or its conjugated complex variable S_{yx} – is the cross-spectrum of the two signals. For the quadratic coherence $\rho^2_{x,y}(\lambda)$ of two signals x and y which have the frequency λ the following relationship applies:

Quadratic coherence:

$$\rho^2_{x,y}(\lambda) = \frac{S_{xy}(\lambda) \cdot S_{yx}(\lambda)}{S_{xx}(\lambda) \cdot S_{yy}(\lambda)} = \frac{\left|S_{xy}(\lambda)\right|^2}{S_{xx}(\lambda) \cdot S_{yy}(\lambda)} \qquad (1)$$

Our approach in this investigation is based upon the coherence of neighbouring signals of the EEG and the evaluation of the synchronicity between EEG signals. The coherence is dimensionless. For the relationship between the negative entropy $E^{neg}_{x,y}$, the transmitted information $-I_{x,y}$, and the quadratic coherence $\rho^2_{x,y}$, Schack has provided the following expression (Schack, 1997, 1999; Grießbach, 1990; Schack, Grießbach und Krause, 1999a, 1999b; Krause, 2000):

Negative entropy:

$$E^{neg}_{x,y} = -I_{x,y} = -\int_0^{\lambda_{Nyq}} \ln\left[1 - \rho^2_{x,y}(\lambda)\right] d\lambda \qquad (2)$$

Thus, by measuring coherence we are indeed also measuring *Information*. Equation (2) characterises the connection between quadratic coherence and negative entropy. This representation of the negative entropy of two EEG signals reflects the postulate that the information transfer during brain activity takes place through oscillatory activity in neural networks.

Normally, calculation of the frequency spectrum S of a time function, as needed for the determination of coherence by equation (1), requires a certain time; this time depends upon the lower limit of the frequency of the time function. With a lower frequency limit of 1 Hz the time required to register the frequency properties of the time signal is one second. In this time needed for calculating the frequency spectrum, the cognitive process cannot be registered. However, as cognitive processes take place on a time-scale of milliseconds, the use of a method of this kind for coherence analysis in studying cognitive processes is ruled out.

Schack *et al*. (1999) determined the negative entropy from EEG signals for a concept-activation experiment in the running process on the basis of a momentary coherence calculation. The experiments for this were conducted by Nicole Kotkamp (1998). In a concept-activation experiment performed according to Posner and Mitchell (1967), the subjects were shown two letters, for example a capital “A” and a small “a”, separated by a given time interval, and were asked to decide whether the two letters had the same name. For this concept-activation process Posner and Mitchell measured a time of 658 ms at the behavioural level. This interval also included the time required to press a button, so that the actual concept-activation time must have been much shorter. It is clear that a classical coherence analysis, requiring for example 1 second for the calculation of a value, cannot represent this concept-activation process.

In the experiment described above, the momentary coherence was calculated according to the approach introduced by Schack (1997), which allows a high resolution of time and frequency. This approach rests upon general *adaptive* principles (Grießbach, 1990). In particular, it is based upon the fitting of a bivariate linear model (the ARMA model) with time-dependent parameters and the parametric calculation of the spectral density matrix at every time point *n*. Thus a parameter estimation from the “resting” EEG before the actual concept-activation process is carried out, and this estimation provides the basis of the calculation of the auto- and cross-performance spectra – already at the commencement of measurement of the cognitive process that is to be followed. During the measurement, the parameters are corrected. From the momentary spectral density matrix Schack determined two signals, $\{x_1(n)\}$ and $\{x_2(n)\}$, their momentary autoperformance spectra $S_{1,1}(n,\lambda)$ and $S_{2,2}(n,\lambda)$ and cross-performance spectrum $S_{1,2}(n,\lambda)$ and thus the momentary coherence spectrum $\rho^2_{1,2}(n,\lambda)$:

Momentary coherence spectrum:

$$\rho^2_{1,2}(n,\lambda) = \frac{\left|S_{12}(n,\lambda)\right|^2}{S_{11}(n,\lambda) \cdot S_{22}(n,\lambda)} \,. \qquad (3)$$

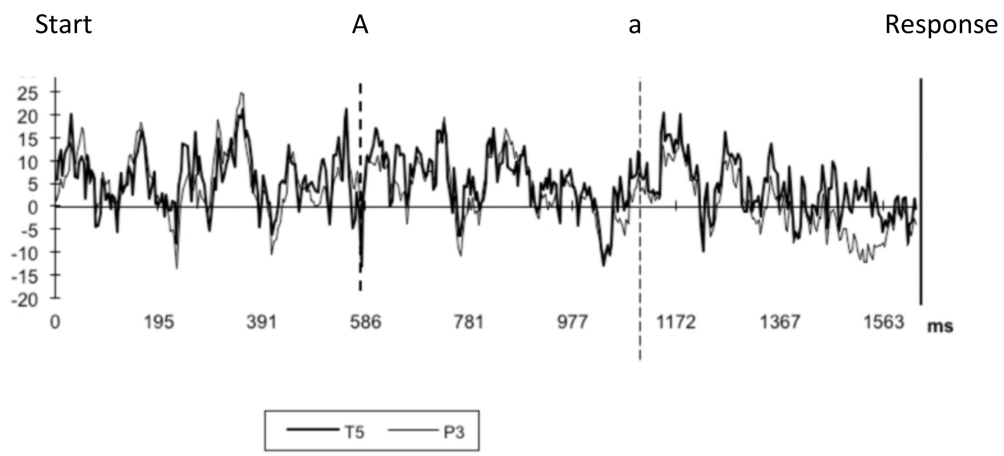
From this, the time-dependence of the negative entropy $E^{neg}_{n;x,y}\left[\lambda_1, \lambda_2\right]$ with respect to a frequency band $\left[\lambda_1, \lambda_2\right]$ can be calculated:

Momentary negative entropy:

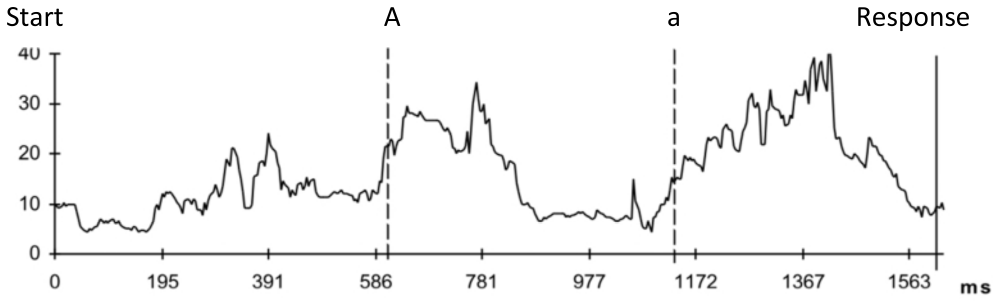
$$E^{neg}_{n;x,y}\left[\lambda_1, \lambda_2\right] = - \sum_{\lambda_1 \le \lambda_k \le \lambda_2} \ln\left[1 - \rho^2_{1,2}(n,\lambda_k)\right]\Delta\lambda \;\; \text{mit} \;\; \Delta\lambda = \lambda_{k+1} - \lambda_k \,, \;\; n = 1,\ldots, N \,. \qquad (4)$$

The results of this calculation are shown in Figure 2.

EEG curves measured at the electrode pair T5/P3 during concept activation



Momentary negative entropy (13–20 Hz) at T5/P3 during concept activation



Momentary coherence (13–20 Hz) at T5/P3 during concept activation

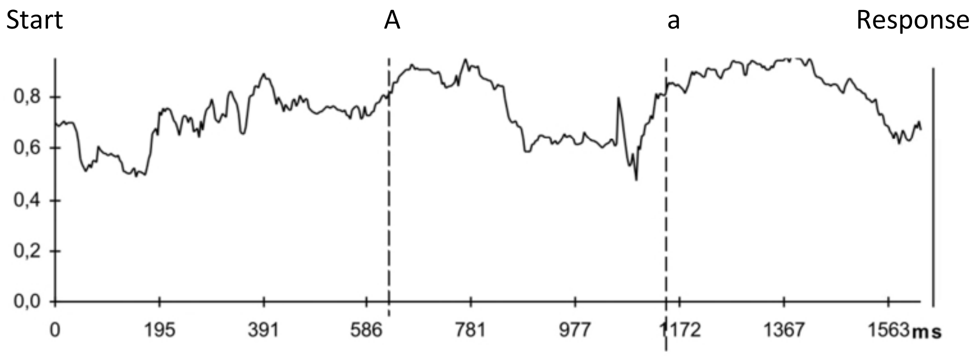


Figure 2. EEG signals, momentary negative entropy and momentary coherence as functions of time in a concept-activation experiment (Schack 1999). At the two time points indicated, an upper-case letter (A) and a lower-case letter (a) were shown and the subject had to decide whether they had the same name. The activation following a name (here at the electrode pair T5/P3) was considered to be related to the increase in negative entropy and in coherence. The electrode pair T5/P3 is located above the Broca centre, which is associated with concept-processing. A detailed time–frequency analysis of the coherence function revealed an especial sensitivity of the frequency region 13–20 Hz in connection with these elementary cognitive processes (Schack & Krause, 1995). (From W. Krause (2000), Reproduced by kind permission of the publisher).

The higher negative entropy and the greater coherence above the concept-activation centre (T5/P3) immediately after the appearance of the letters could suggest activation of the concept of “A” and “a”, and could be an expression of the sensitivity of this information variable in the cognitive process. However, at this point that would be pure speculation. We shall return to this in Part 2.

The use of the coherence measure permits the cognitive information-processing operation to be projected onto the synchronicity of two signals, and this is a representation onto a different variable from amplitude. It has not yet been shown strictly (mathematically) what connection there is between the coherence in EEG, determined in this way, and the binding problem in brain research (von der Malsburg, 1994; Engel, Brecht, Fries und Singer, 1998); however, it seems very probable that a connection exists between EEG coherence and cortical synchronisation at the neuron level.

It is plausible that information must be measured by an information variable, just as performance is measured by a performance variable. Against this background it first appears implausible that information-processing can also be represented by a performance variable, although this has been shown many times by GFP or by the evoked potential (EP) in the study of human information-processing. This apparent contradiction is however easily resolved, when one considers how the recording of electrical activity is done. In classical EEG measurements the electrodes are distributed over the head in such a way that each electrode covers an area approximately 2.5 cm diameter. The amplitude of the electrical signal as measured by each electrode at the surface of the head arises as a sum of the electrical activities of a large number of neurons and their associated circuitry. If we assume the presence of 10^{10} neurons and 20 electrodes, then each electrode will be associated with an order of magnitude of more than 5×10^8 neurons plus circuitry – a network of sufficient size to bring about a great deal of superposition. The result of such overlapping will depend upon the phases of the individual signals: the overall amplitude of the superposed signals at the electrode will be greatest, when the individual signals of the neurons are in phase, i.e., when the degree of synchronicity (the coherence) is high. Against this background it becomes plausible that information is reflected in a performance variable: this kind of measurement is associated with a high amplitude which has a high degree of synchronicity, that is, a high coherence. If information is reflected in coherence, then in this kind of measurement it must also be reflected in the amplitude.

In summary: There is a functional relationship between coherence and information. Through coherence, information can be measured (equation (2)). Adaptive coherence (Schack, 1997) allows a process analysis on the time-scale of milliseconds. This opens up completely new possibilities for the measurement of cognitive processes.

It should be noted that we are – of course – here considering only Shannon information. Whatever meaning this information may carry we are not taking into consideration.

Part 2: Topology

2.1 Objective

In the 5000-year history of mathematics, the change in representation of tasks has always played an important part. Thus, for example, the number five can be represented

– by a set of five real stones, or

– by various conventional symbols for five objects: the word “five”, or the character “5” or “V”. (Zimmermann, 2003). Similarly, a circle can be represented as a figure or as an equation.

Change in representation also plays an important part in the psychology of memory. Engelkamp (1990) entitled his book *Das multimodale Gedächtnis* (The Multimodal Memory); in it, he concentrated on the fact that things are remembered better when they are present in several modalities. This is termed the *multimodality hypothesis*. An object, shown as a picture and in words, is more reliably reproduced that when it is presented only as a picture or only in words.

In the psychology of thought, and especially in research into highly gifted people, verbalisation is used to show that individuals with a strong talent for mathematics generate internally several representational forms of a task. For example, they conceive of a circle both as a picture and as the equation for a circle. This finding, at the level of behaviour (or action) also finds correspondence at the neurological level; in persons with a strong mathematical talent, both sets of cortical networks are activated: the ones for formulae and the ones for pictorial-descriptive conception (Krause, Seidel und Heinrich, 2003a; Krause, 2006). So far, so good. Yet this view has frequently – and rightly – been criticised because of its ambiguity. To meet this criticism, we have broadened this standard method by adding functional dependence, and we show that the introduction of a “psychophysical function” reduces the ambiguity of the topological assertion.

However, at the action level, the description is much more sophisticated: Heinrich (2003a, 2003b, 2003c) speaks of the interaction between *calculation* and *pictorial conception* in the solution of mathematical problems, and Klix (1992) asserts that highly talented individuals are able to choose suitable solution strategies for given problems in such a way that they fit “like the key to a lock”. Such descriptions at the action level can be used to characterise not only representations, but also solution processes; their equivalents at the neurological level have not yet been found. Here we used the microstate-sequences approach, based upon the microstates found by Lehmann (Lehmann *et al.*

1987) in electroencephalograms, and we present ideas and preliminary results in the hope of stimulating further investigations. This will be found in Part 3.

2.2 Why a topological analysis?

The performance of mathematically highly gifted people has always been fascinating, and there has been no lack of attempts to explain it. For example, a mathematically talented person presented with the question

> "How many diagonals has a polygon with 23 sides?"

solved it in six seconds (Heinrich, 1997). An attempt at explaining this invoked the multimodality hypothesis, which posits that mathematically highly talented individuals internally conceive the "basic building-blocks of mathematical problems" in two modalities (e.g. a circle and the equation for a circle) and, according to the effort required to solve a problem, approach it in one or other of these modalities, while persons with a normal mathematical ability activate preferentially only one modality. A hypothesis of this kind can be verified by the observation that in mathematically highly talented persons the modalities that are additionally active correlate with additionally activated cortical areas in the neural network. If it can be shown at the action level that the picture of a circle and the equation of a circle are activated simultaneously, then the cortical areas for "calculating" and for "picture-processing" are also activated simultaneously. Let us clarify this basic idea of topological analysis by using as examples the mathematical problems chosen by Heinrich (1997). Figure 3 shows the tasks used. In the figure, the category "single-modality strategy" refers to the use of only a single modality strategy (calculation or picture-processing) for solving the task, and "dual-modality strategy" to the possibility of attacking one and the same problem in either of two ways, such as solving an equation ("calculation") or by geometric transformations ("picture-processing"). The expression "by calculation" we sometimes replace by the word "conceptual", because this has entered into general use in the psychology of memory as the opposite of the term "pictorial".
In Figure 3, the problems associated with single-modality strategies serve as reference tasks: the addition of single-digit numbers for calculation and mental navigation for picture-processing. The latter task involves deciding whether moving according to the arrows shown will lead from a starting point at the lower front left to the given point at the upper rear left. Such mental navigation takes place within a "pictorially" visible conception. In the lower part of Figure 3 three tasks are shown which can each be solved by either modality strategy. The task on the left can be solved by Pythagoras' theorem (calculation) or by placing a second diagonal in the square and turning each of the resulting four triangles over to the outside (pictorial). The middle task can be solved by counting (calculation) or by displacement (pictorial). For the problem set at the beginning of this section concerning the 23-sided polygon, one of our mathematically gifted subjects, as mentioned above, required 6 seconds. If an inductive process had been followed (calculation), going from a square to a pentagon, a hexagon and so forth to a 23-sided polygon, then this would hardly have been possible in 6 seconds. On being asked, the subject said that he had imagined one corner and the 22 lines going out from it, counted the diagonals (20) divided by 2 (10) and finally multiplied this by the number of vertices.

If it is correct, this phenomenological interpretation at the behavioural level would also be expected to correspond to a topological interpretation at the neurological level: If we proceed from the assumption that in solving "dual-modality tasks" mathematically highly gifted persons use two modalities and normally gifted persons use only one (at least at the beginning of the solution process), then a topological analysis at the neurological level should show two regions of cortical activity in mathematically highly gifted individuals and only one region in mathematically normally gifted people. This might allow a verification of the modality hypothesis. How can it be demonstrated?

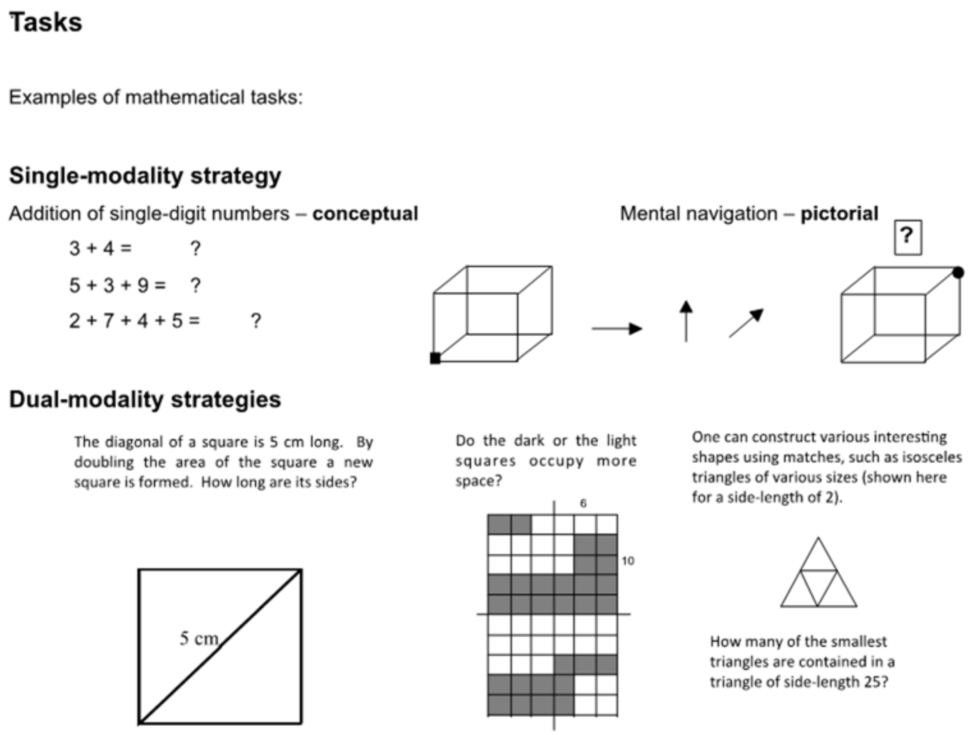


Figure 3. The mathematical tasks used in studies, collated by Heinrich (1997). For explanation, see text.

2.3 The traditional procedure

The substitution (or subtraction) method invented by Donders (1889) allows the determination of the time required for a cognitive operation. To do this, one determines the difference in time required for the solution of two tasks that differ in precisely one cognitive operation. This presupposes that these tasks are identical in respect of the other cognitive operations involved.

In the work described here, this difference method (now widely used in the neurosciences) was applied separately for each subject. For example, we can determine the topology for a mathematically highly gifted subject solving a dual-modality problem (“conceptual” and “pictorial”; e.g., the problem on the lower left in Figure 1) and then subtract from that the topology of the same subject solving a single-modality problem (“conceptual”; e.g., the addition of three and four numbers in Figure 1). The difference between these should then reveal which cortical area is responsible for processing the image, for example as we showed above for turning over the triangles. In contrast, for mathematically normally gifted subjects the difference should not reveal any additional activation, as for these persons being presented with a dual-modality problem should only activate one strategy – in most cases, they start off by calculating. For the example above, they might use Pythagoras' theorem. This naturally presupposes that the same cortical area is activated in solving the addition problem and in applying Pythagoras' theorem. The topology was determined by using coherence maps derived from electroencephalograms.

The EEG assessments were performed with the adaptive EEG coherence analysis developed by Schack (Schack, 1997, 1999; Schack *et al*. 1995, 1999a, 1999b). The coherence is a measure of the synchronicity of two signals. High coherence means high (synchronous) activation. Observations were restricted to neighbouring electrode pairs and to the usual frequency band of 13–20 Hz. For details of the EEG evaluation methods the reader is referred to the literature cited. The time-frame for the analysis was restricted to the first second after the subject had understood the instructions.

The starting point of 800 ms (rather than 0 ms) was chosen because of the methodological conditions of the adaptive EEG coherence analysis. The analysis interval of 1200 ms was chosen arbitrarily, partly because of the need to process very large datasets. Figure 4 shows the difference coherence maps obtained for normally and highly gifted subjects solving mathematical tasks (a dual-modality strategy, 2M, minus a single-modality strategy, 1M).

The observation in Figure 4 implies that within the first second after the instruction has been understood the brains of highly gifted subjects were activated for both of the relevant modalities while, in contrast, the brains of normally gifted subjects in the same time-frame of one second after understanding the instruction did not (yet) have access to a "pictorial" modality strategy.

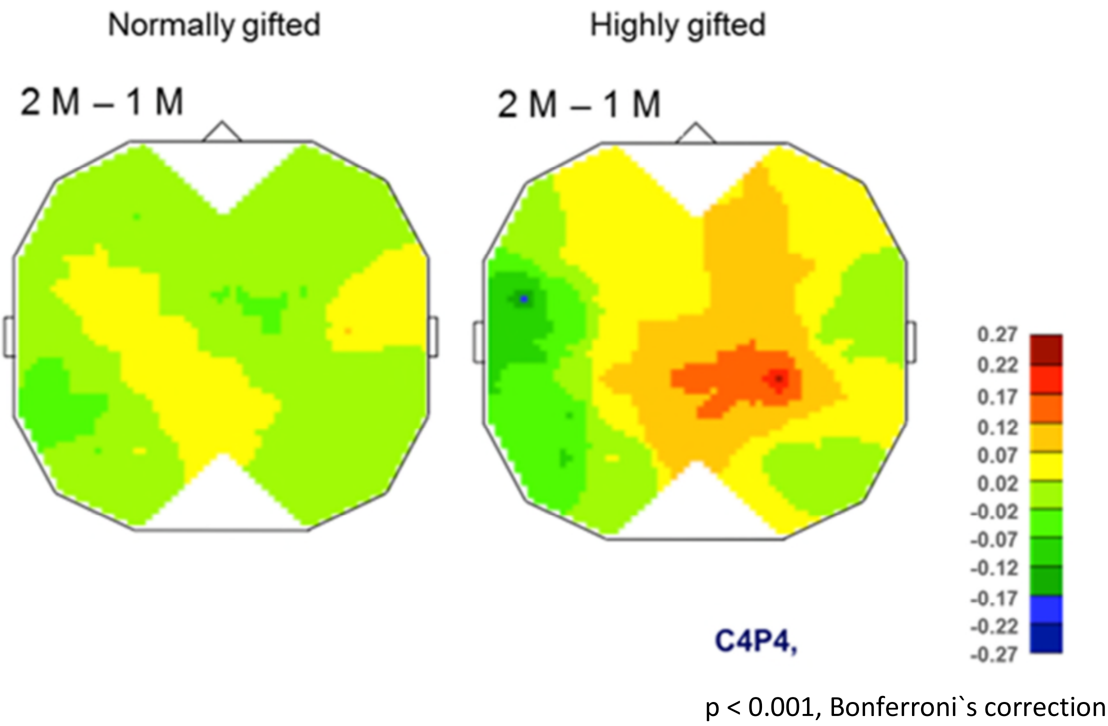
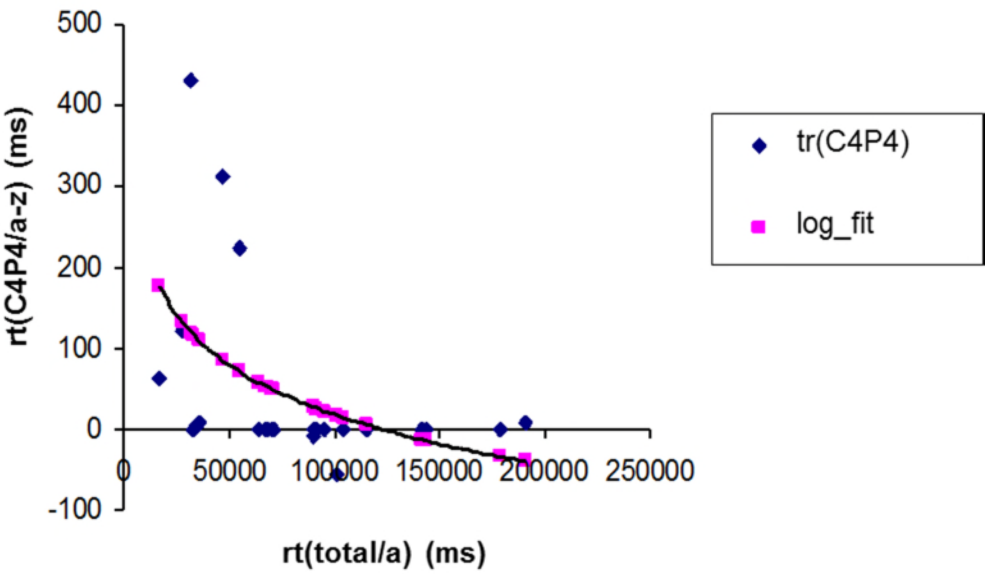


Figure 4. Averaged difference coherence maps during the solution of mathematical tasks (a dual-modality strategy, 2M, minus a single-modality strategy, 1M) for normally and highly mathematically gifted subjects. The coherence difference of 0.27 across the electrode pair C4P4 (the red centroparietal area to the right) is significant (p < 0.001, using Bonferroni's correction). The scale of coherence values runs from +0.30 (dark red) to –0.30 (dark blue). Each of the 14 normally and highly gifted subjects was assessed individually. For 12 out of 14 highly gifted individuals there was a significant difference across the electrode pair C4P4. The map was derived by averaging over these 12 highly gifted subjects. For the normally gifted subjects no difference was found. (The figure is taken from Krause, Seidel and Heinrich, 2003, by kind permission of the Erhard Friedrich Verlag, Seelze.)

This result may nonetheless be questioned, because Donders' condition (given above) is not strictly fulfilled. It is true that the two selected tasks (a "single-modality" versus a "dual-modality" strategy) differ in respect of precisely one modality. However, it has not been shown rigorously that the cognitive operations in the common modality are completely identical, as required by Donders. In the terms of our example: it remains an open question whether the summation of numbers is, cognitively and neuronally, equivalent to the operations performed in applying Pythagoras' theorem. We therefore wish to tighten up the assessment, by making an additional requirement: the demonstration that the results at the action level depend functionally upon the results at the neurological level.

2.4 Determination of a "psychophysical" function between the action and neurological levels

We begin with a plausibility consideration. The mathematical tasks set out in Figure 3 were chosen by Heinrich (1997) such that the use of a pictorial modality strategy would shorten the solving time in comparison with a computational strategy. To formulate a hypothesis about a function that relates the action level and the neurological level, we consider two extreme cases: (1) If a pictorial strategy is followed, then the activation time for the cortical area responsible for image-processing should be long and the solution time should be short. (2) If a pictorial strategy is not followed, then the activation time over the cortical area for image-processing should be short (or zero) and the solution time should be long. A "psychophysical function" that represents the activation time for the cortical area for image-processing as a function of the solution time should therefore show a negative slope, while no functional relationship would be expected over any of the other cortical areas. This presupposes calculation of the difference between a dual-modality task (denoted with an "a" in Figures 5 and 6) and a single-modality task (denoted with a "z" in Figures 5 and 6). Figure 5 shows a right centroparietal "psychophysical function" obtained across the electrode pair C4P4.



$R^2 = 0.22$	$F = 5.9$	sgn $F = 0.023$
beta = -0.47	$T = -2.44$	$p = 0.023$
const. = 1047	$T = 2.56$	$p = 0.018$

Figure 5. Difference in activation time rt(C4P4/a–z) in the solving of dual-modality problems (a) minus single-modality problems (z) across the electrode pair C4P4 as a function of solution time rt(total/a) for dual-modality problems. Each dark-blue point (tr(C4P4)) in the diagram stands for time difference measured for a single subject. (From Krause, Seidel and Heinrich, 2003; by kind permission of the publisher Erhard Friedrich Verlag, Seelze.)

The negative slope of the fitted curve is significantly different from zero (B = –79.6; $p = 0.038$). Figure 6 shows the "psychophysical function" for all electrode pairs.

The result corresponds to expectation. The "psychophysical function", which represents the activation time across the electrode pair C4P4 as a function of the solution time, shows an relationship of inverse proportionality for this time interval (the first second). The longer this selected cortical area (in the early time-frame selected) is active, the shorter is the solution time.

The function in Figure 6 represents a relationship such as is of importance in understanding the inner and outer psychophysics of cognitive processes (Sommerfeld, 2001). Analogously – even with thought processes – a function can be formulated relating processes that take place internally and externally.
We interpret this finding as follows. An early, longer activation of the cortical areas responsible for a pictorial modality strategy shortens the solution time. This underlines the influence of the early availability of strategies upon the solution time, and it offers a possible explanation for the short solution times achieved by highly gifted individuals.

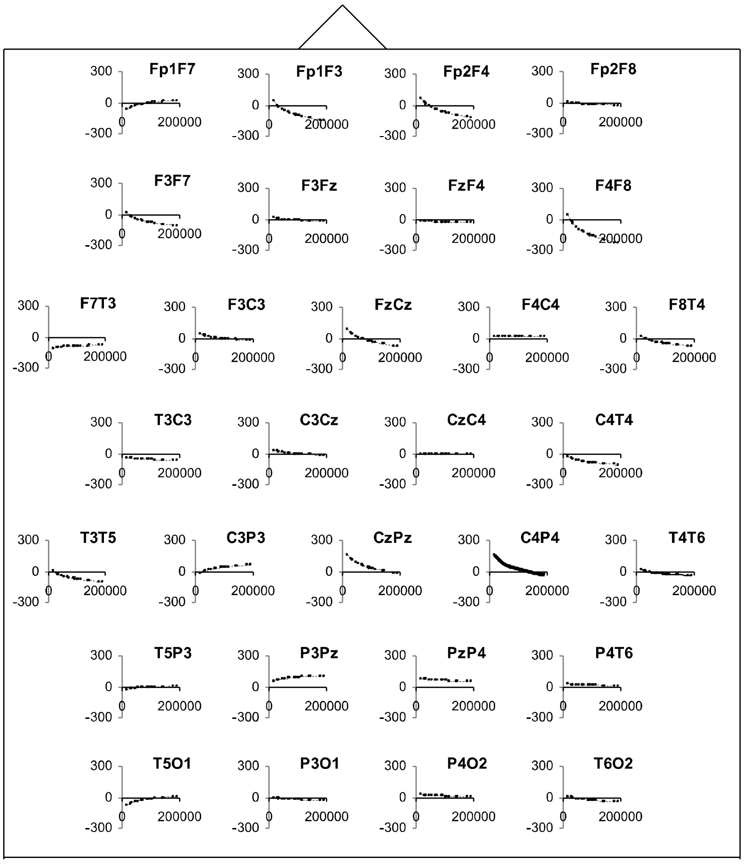


Figure 6. The “psychophysical function” in Figure 5 for all electrode pairs. Only the pair C4/P4 shows a negative slope significantly different from zero. None of the other curve-fits across the electrode pairs shows a systematic relationship between the activation and solution times.

The processing time in Figure 6 is clearly not an independent variable of a single subject. To attain this one would have, for example, to introduce a learning process through which the processing time could be shortened for each individual subject. At present this must remain a topic for future research.

In summary: The existence of a functional relationship between the action level and the neurological level, a “psychophysical function” (Figure 5) can be demonstrated. This supports the multimodality hypothesis.

Part 3: Microstate sequences

3.1 Objective

Measuring mental performance remains a problem. It still needs to be shown that mental performance can be represented in an ordering of microstate sequences and can thus be quantified as entropy decrease. We therefore take up a concept introduced in physics some 150 years ago by Clausius (1865), which was later developed as a measure of disorder: entropy. Microstates (Lehmann, 1987) are activation states in the brain and are stable over time. Microstate sequences are treated as Markov chains and their properties are calculated on a differential basis.

A literature search using the search engines “PsyInfo” and “PubMed”, with the key word “microstates” and spanning the past 25 years, yielded a total of 128 articles. None of these included the measurement of entropy – or entropy decrease – in microstate sequences. This adds force to our aim in this article – the aim of encouraging continued research by this approach.

In this section we proceed as follows. We describe first *microstates* and their properties, and then we consider the projection of cognitive processes upon the properties of the microstates. We then discuss *microstate sequences* and their properties, and describe the projection of cognitive achievement onto sequence properties of microstates.

3.2 Microstates and their properties

For both EEG metrics (cf. Part 1), we consider microstates and sequences of microstates: first their performance and then their coherence.

3.2.1 Microstates on the basis of equipotential contours

One method of analysing EEGs has become known as “segmentation” and was developed by Lehmann and Strick (Lehmann, Ozaki & Pal, 1987; Strick & Lehmann, 1993; Strick, 1993). Lehmann has occasionally referred to it as a method for investigating “atoms of thought”. Such microstates – termed segments – can be determined from map sequences. A map is the distribution of activation over the topography of the head. The basic idea is the following. Any potential distribution measured over the electrode field can, for every point in time, be represented as a map by using equipotential contours. According to a criterion to be defined, the similarity between two consecutive maps is determined. If this similarity criterion lies below a given value, then this demarcates a segment in time. The duration of a segment is then the time between two neighbouring borders. A sequence of similar maps is termed a segment and interpreted as a microstate. (Note that in Section 3.4.1 we also describe how Schack classified the segments in terms of clusters.) One possibility for determining the similarity of two maps consists in finding the positions of “centres of gravity” in the potential distribution, so-called centroids. Two consecutive maps are then regarded as similar if, for example, their centroids do not leave a predefined window.

In each of these time-independent segments, as Lehmann terms them, a certain information-processing operation must be taking place.

By this method, sequences of time-independent elements can be determined for any arbitrary task. Using this method and a software package developed by Strick (1993), Tietze (1996) was able to distinguish conceptual activation from pattern comparison, and to show that the coding of information takes place in distinct segments. We shall look more closely at this in Section 3.3.

3.2.2 Temporal properties of microstates

Tietze (1996) determined the frequency distribution of duration of segments (duration of microstates) using the method of Lehmann and Strick as described in Section 2.1 above. The following consideration of the distribution of segment durations is thus based upon global field power, that is, on performance as a measurable variable. The distribution of segment durations was determined for concept activation (Na_Aa) and pattern comparison (Au_Ga). In addition, the histogram of segment durations was determined for two further comparison experiments: Na_AA and Au_AA. In the first experiment subjects were asked whether the two letters had the same name. This challenge can obviously be met without concept activation. The second experiment was a comparison of letters, which likewise does not require concept activation (for the experiment, see also Part 1).

The distribution of segment durations for the frequency band 2–30 Hz is shown in Figure 7. Figure 8 shows the histogram for frequency bands of various degrees of narrowness. The

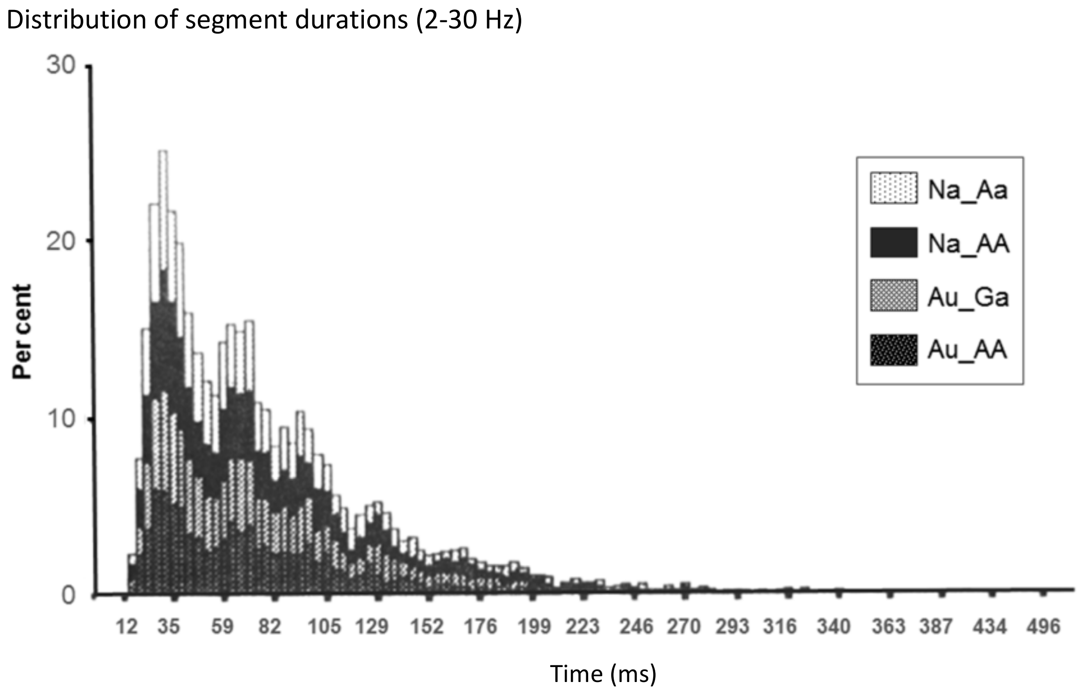


Figure 7. Relative frequency (%) of segment durations as a function of time for the four conditions Na_Aa, Na_AA, Au_Ga and Au_AA in the frequency band 2–30 Hz (Tietze, 1996). (From Krause, 1997. Reproduced by kind permission of the publisher.)

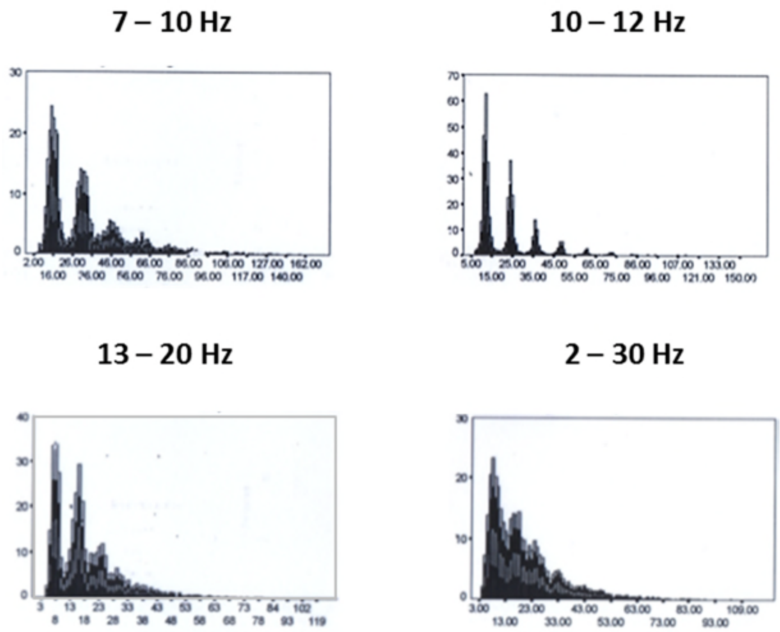


Figure 8. Relative frequency (%) of segment durations for four frequency bands, analogous to Figure 7 (Tietze, 1996). The abscissae show the number of frames (sampling points). A frame represents 3.91 ms.

challenges Na_Aa, Na_AA, Au_Ga and Au_AA are parameters. The distribution of segment durations is almost discrete for each of the four challenges. It is clear that the discrete distributions with narrower frequency bandwidths must become sharper. Figure 8 underlines clearly this increase in discreteness. The maxima of the distribution in Figure 7 are found preferentially at 32 ms and integral multiples thereof: 64 ms, 96 ms, 144 ms, and so on. The shortest interval measured here, 32 ms, is indeed seven times higher than Geißler's time-quantum of 4.56 ms (1991, 1992, 1994); however, any absolute comparison of the discrete durations as determined by EEG with Geißler's time-quantum hypothesis should be made with caution, as the segment durations are here determined by the restrictions introduced (see the similarity criteria set out in Section 3.2.1). Nevertheless, this finding emphasises the coupling between the action level, at which Geißler obtained his data, and the neurological level. At both levels, a discretisation of elementary cognitive processes takes place.

According to this finding by Tietze, the segments that are stable in time last for 32 ms or an integral multiple thereof. The significance of this for the mechanism of information-processing should be made clear at this point: In discrete time intervals the activation is constant over the entire topography. In these “constant” time intervals information is processed, and the coding of information takes place. Can this be demonstrated, and can one point to particular segment durations (microstate durations) that are associated with this?

3.3 Mapping of cognitive processes to properties of microstates: coding

Here we examine in more detail segment duration and the positions of highly active centres. By the method of segmentation, the average position of the centre of gravity (centroid) can be determined for each segment. In this way the segments are characterised by two properties: their duration and the position of their centroid in the electric field. To define their location, a five-by-five co-ordinate system is used, which is defined by the 10-20 system of electrode placing. The co-ordinate pair (1,1) represents the position “left frontal”, (4,4) represents “right temporoparietal”, and (3,3) represents the position of the electrode Cz (in the centre). As the distribution of electrical potential can adopt both positive and negative values, two centroids are determined which together define the position of a dipole. Tietze initially averaged the positions of the centroids for each segment and for the two sets of conditions ‘concept activation Na_Aa’ and ‘point-pattern comparison Au_Ga’, averaged over all segment durations. He found the expected left-displacement of the centroids in the 3rd and 4th segments when concept activation Na_Aa occurred, compared with pattern comparison Au_Ga. In contrast, no significant change was observed in other segments, either before or after concept activation. The observed left-displacement (observed in the left hemisphere, because of the linguistic processing) corresponded to expectation. However, the small magnitude (0.3) of the significant differences in location of the centroids was unsatisfactory, so that a re-analysis appeared to be called for.

On the basis of the discrete distribution of the segment durations, Tietze conducted a second assessment in which the segment duration was restricted to 94 ms (this corresponds to the third maximum in the distribution of segment durations according to Figure 7, ranging from 78 ms to 110 ms). Details of the method (material, experimental procedures, sample size, data acquisition and evaluation) can be found n the publication by Tietze (1996; cf. also Krause, 1997). Figure 9 shows the result. Significant differences were found between the centroids' positions in the 2nd and the 4th segment when the two sets of conditions ‘concept activation’ and ‘pattern comparison’ were compared. It is possible that the concept activation begins with the 2nd segment and is complete at the latest in the 4th segment. In comparison with the averaging over *all* segment durations, as described above, the consideration of only *one single* segment duration of 94 ms shows that the distance between the centroids' positions under the two sets of conditions has become substantially greater. The centroids for concept activation have moved further over to the left.

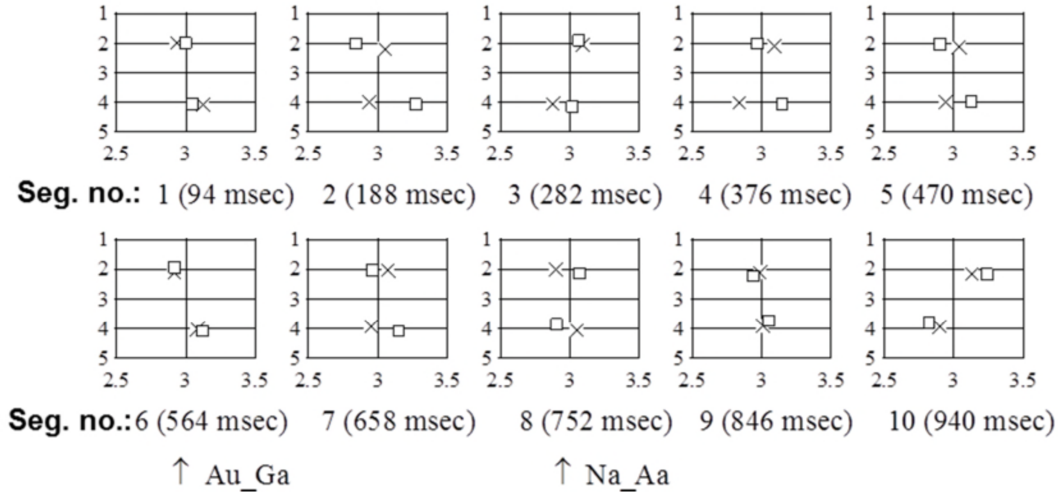


Figure 9. Centroid positions in consecutive segments during concept activation (Na_Aa) and point-pattern comparison (Au_Ga) with a segment duration of 94 ms (after Tietze, 1996). The co-ordinates have values between 1 and 5 in each of the x and y dimensions. The co-ordinate pair (1,1) denotes a left-frontal area and (5,5) a right-temporoparietal area. The x and y co-ordinates are not equidistant. The symbol × stands for concept activation, the square stands for pattern comparison and the arrows show the time of response. (From W. Krause, 2000, reproduced by kind permission of the publisher).

It is obvious that the difference between the two sets of conditions becomes more clearly apparent in this evaluation, which takes into consideration only the 94 ms segment duration. Thus, shorter segment durations may perhaps make only a small contribution to the information-processing operation. It is not yet possible to make any statement about the longer segment durations. In other words: The activation of a concept requires at least one segment, with a duration of at least ca. 100 ms. Shorter segment durations are apparently inadequate for this kind of semantic coding. It is clearly necessary to test these findings more thoroughly.

At the action level, Klix (1992) and van der Meer and Schmidt (1992) found a minimum recognition time of 226 ms for establishing a difference in characteristics when comparing pairs of concepts. If one generalises from this time, and posits it as the time required for a cognitive operation at the action level in concept activation, then clearly at least two segments – with a segment duration of ca. 100 ms at the neurological level – are required to fill out this interval. Can it be that this doubling is related to the increase in reliability of transmission in human information-processing?

In spite of high pulse frequencies at both the action level (Geißler) and the neurological level (Tietze), a certain minimum time is needed for coding in concept activation and pattern comparison, at both levels.

Tietze also carried out this assessment of the segmentation and centroid-location for two comparable processes the do not require concept activation: letter comparison (Au_AA) and a "non-concept-activation" using identical letters (Na_AA). For these two sets of conditions the centroids are expected not to differ in the dorsal region, as the both tasks can be performed without concept activation. This was indeed found to be the case.

To summarise: To code semantic information – here concept activation – segments (microstates) of long duration are required – here at least ca. 100 ms. Segments of shorter duration are apparently much less frequently responsible for this kind of coding. Thus, convincing evidence can be adduced that semantic information is processed in the microstates, although this still requires further investigation.

3.4 Microstate sequences and their properties

3.4.1 Number of microstates, alphabet

Information transfer between neurons, or between cohorts of cells in a neural network, is clearly associated with greater synchronous oscillation of the electrical signals that are measured over these cortical areas. Using the adaptive signal analysis introduced by Grießbach (1990) and the adaptive EEG coherence analysis developed by Schack (1997, 1999), Schack *et al*. showed (1999a,1999b,1999c) that temporally stable intervals can be observed in EEG measurements even when instead of amplitude the adaptive coherence is used as the basis of the signal analysis. The EEG is recorded by using the classical placing of 10–20 electrodes. From this, the coherence is calculated for each pair of neighbouring electrodes and likewise displayed as a function of time. At any arbitrary point in time (here every 4 ms) the coherence values for all 30 electrode pairs can be extracted and represented as a map covering the topography of the head, using colour contours. The time course of this (not shown here; available in the publications referred to below) shows that temporally stable intervals arise, as already found by Lehmann in amplitude measurements. These temporally stable intervals we term coherence segments, or simply segments, with their corresponding respective segment durations (cf. Section 3.2.1). The unchanging coherence over the entire topography for some intervals expresses the processing of information, insofar as information-processing can be attributed to oscillation of signals (treated in full by Schack, 1999, and Sommerfeld and Krause, 2013).

Schack (1999) classified the segments by clustering them according to their topographic similarity; she obtained in this way coherence maps – analogous to Lehmann's considerations – which she terms “microstates”. The number of microstates is generally determined during the clustering by applying a “penalty function”, which must attain the lowest possible value. Detailed accounts of the procedure are given by Schack (1999) and Seidel (2004). Seidel has used this evaluation method to perform experiments on mathematical problem-solving, and determined the number of microstates that must be distinguished (Seidel, 2004; Krause, Seidel und Heinrich, 2003). To remind the reader: a microstate represents a distribution of activation across the head, measured in this case by adaptive EEG coherence. One might infer from this that the number of microstates to be distinguished is very large, because of the great complexity of the brain – not only on account of the number of its neurones and their interconnections, but also on account of the its architecture and dynamics. However, using the method of Schack and the mathematical problem-tasks developed by Heinrich (1997), Seidel was able to show that this is not the case. Rather, there are only a *few microstates* that can be distinguished by experiment. According to the choice of criterion for the penalty function, Seidel was able to discern between six and ten microstates that were specific to individual subjects for a class of tasks. Theoretically, this method could allow the distinction of 300 microstates (assuming 30 points of measurement and 10 possible amplitudes at each point).

Kindler *et al*. (2011) and Schlegel *et al*. (2011) both report that their studies revealed four microstates, albeit measured by “global field power”. Moreover, in their study of acoustic hallucination in schizophrenic patients, Kindler *et al*. (2011) also identified the microstate that reflects the acoustic hallucination. However, at present that must be regarded as a special case.

Any arbitrary cognitive process can be represented as a sequence of such microstates. If one imagines the microstates as letters of an alphabet, the cognitive processes are “words” written in that alphabet. Neither the exact extent of the alphabet nor the interpretation of the microstates – that is, the meaning of the letters – has been investigated until now. This will be a field of research for the future.

It is nonetheless possible to place *sub*sequenc*e*s appearing within the microstate sequences at the neurological level within the context of problem-solving behaviour (see Section 3.5.3).

3.4.2 Concatenation, entropy, entropy decrease, order

We first consider – in a casuistic sense – sections from two microstate sequences obtained with one mathematically normally gifted individual and another, highly gifted, individual; the microsequences are measured within the first 10 seconds of each subject's starting to solve a mathematical problem:

Microstate sequences:

– Normally gifted:

. . . . E D A B D A D E F A B D A E B E D A C A B D A D C A F C . . . .

– Highly gifted:

. . . . B E B E B E C E B E B C B E B E B E D C D E F E B E B E . . . .

The six microstates are here denoted A to F. For demonstration purposes (only), the same notation is used for the microstates in both persons. As described above, the microstates are in fact individually specific, but nevertheless they are similar between individuals. Phenomenologically, one immediately observes longer chains of the same microstate in the highly gifted subject and shorter ones in the normally gifted person. By calculating the entropy (or entropy decrease) the creation of order in thought can be quantified (Krause, 1991).

We now consider the entropy decrease associated with these processes. If the microstates are mutually dependent, then the increase in information on moving from one state to its immediate successor is defined by the following relation (Schack, 1999):

$$H_{red} = H - \sum_{i=1}^{N} P(i) \cdot H(i) \qquad i = 1, \ldots, N \text{ (set of microstates)} \qquad (5)$$

The *entropy decrease* reflects the *sequential* property of the microstate sequences. With

$$H = -\sum_{j=1}^{N} P(j) \cdot ld(P(j)) \qquad j = 1, \ldots, N \text{ (set of microstates)} \qquad (6)$$

$$H(i) = -\sum_{j=1}^{N} P(j/i) \cdot ld(P(j/i)) \qquad i = 1, \ldots, N \text{ (set of microstates)} \qquad (7)$$

Thus entropy decrease expresses numerically the decrease in disorder, or the creation of order, in a microstate sequence. With N = 6 microstates its value lies between 0 and 2.59. The entropy decrease is the greater, the more strongly the transition probabilities deviate from random values. Thus an increase in this figure expresses a transition from a relatively stochastic to a relatively deterministic process, a change that we can imagine as the creation of order.

3.5 Representation of cognitive processes on sequence properties of microstates

3.5.1 Entropy decrease

Seidel (2004) used the metric developed by Schack (1999) to measure the entropy decrease that accompanies the solution of mathematical problems by (mathematically) normally and highly gifted individuals. Figure 10 shows a significantly *greater* entropy decrease in highly gifted than in normally gifted persons within the first seconds of problem-solving (the mathematical tasks used have been described in Part 2). This can be regarded interpretatively as the engendering of greater order in the thought process.

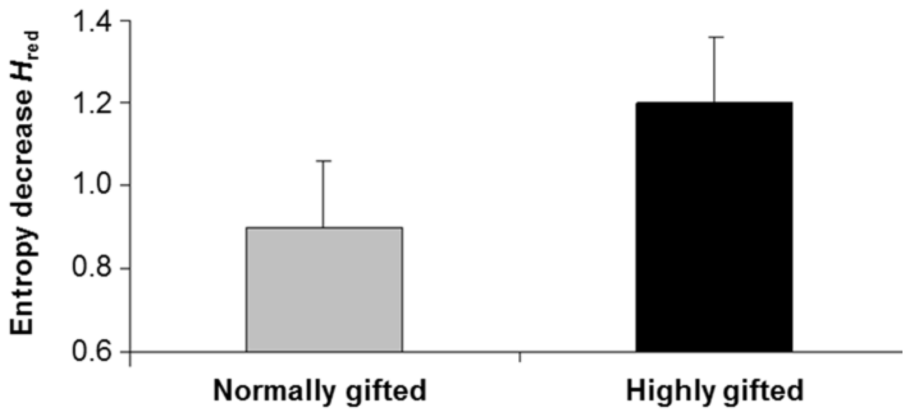


Figure 10. Entropy decrease H_{red} (see equation 5) during the first 10 seconds of solving a mathematical problem, for mathematically normally and highly gifted persons, according to Seidel (2004). Entropy change was determined by EEG coherence analysis by the method of Schack (1999). The difference is statistically significant. The procedure introduced by Schack to register microstate sequences from EEG coherence analysis is described in Figures 16 to 19 of Sommerfeld und Krause (2013).

How can this finding be explained? The results refer to tasks solvable by a dual-modality strategy – that is, tasks that can advantageously be approached by the activation of two modalities (picture and formula). It is thus possible that such multimodal strategies are activated by highly gifted persons from the beginning, when they are presented with a task of that kind. This consideration inevitably leads to the expectation that solving a single-modality task should reveal no difference in entropy decrease between these two groups of persons. In solving a single-modality task, only one strategy is used: either only a formula (e.g., simple addition of numbers) or only a picture. Figure 11 shows the confirmation of this.

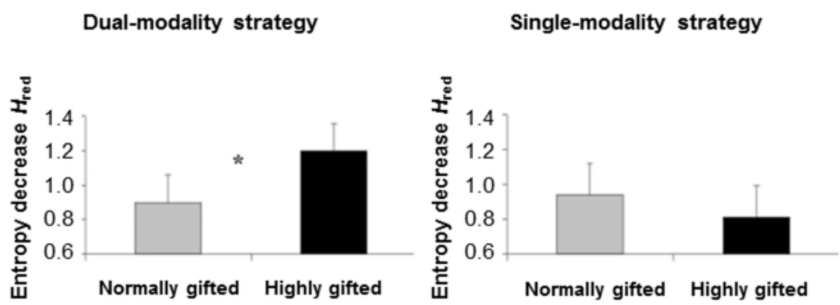


Figure 11. Entropy decrease H_{red} during the first 10 seconds of solving single- and dual-modality tasks, for mathematically normally and highly gifted persons. The difference in the case of the dual-strategy task is statistically significant, while for single-strategy tasks it is not (Seidel, 2004; cf. also Figure 10).

Naturally, it would be expected that for mathematically gifted persons their innate ability to resort to multimodality should be independent of the task. To show this, we compare the entropy decrease for each subject during the solution of simple (cf. Figure 3 in Part 2) and harder dual-modality ones (e.g. "How many diagonals has a 23-sided polygon?"; Heinrich, 1997). For *both* classes of task (simple and hard ones), dual-modality strategies should enable highly gifted persons to show a greater entropy decrease than normally gifted ones do. On the other hand, for various unimodal tasks (e.g., addition versus mental navigation, here referred to as pictorial tasks) the two populations would not be expected to differ. Figures 12 and 13 show the result.

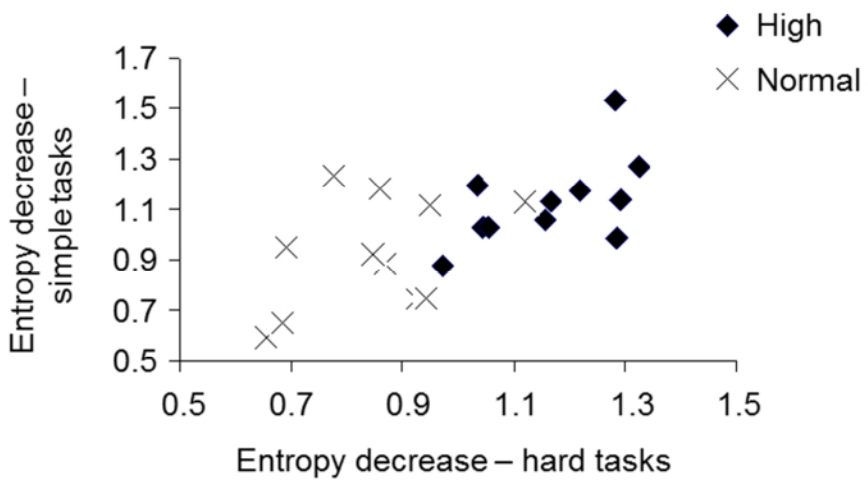


Figure 12. Entropy decrease H_{red} for individual subjects in the two groups (mathematically highly (High) and normally (Normal) gifted persons) during the solution of simple and hard dual-modality tasks. See also Figure 10.

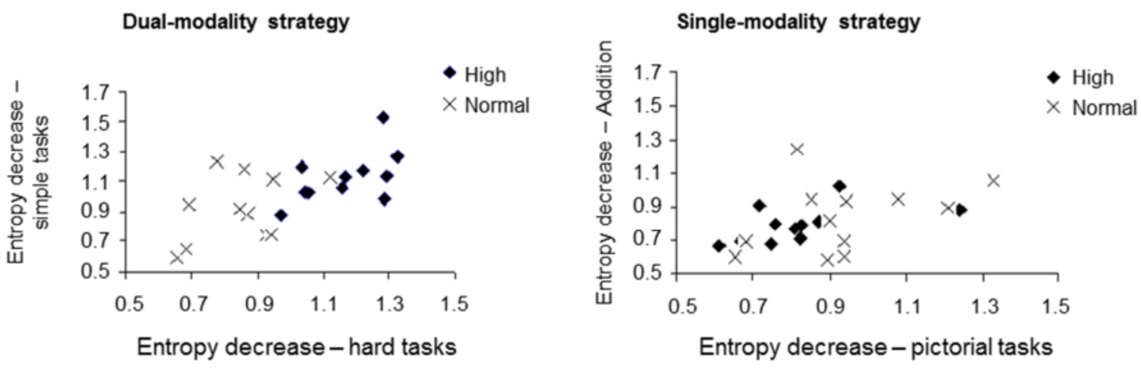


Figure 13. Plots analogous to Figure 12 for single- and dual-modality tasks. As expected, the difference between the subject groups is no longer present in the right-hand plot.

3.5.2 Grammar

To make clear the degree of dependence associated with microstate sequences in the thinking of highly gifted persons, we attempt to imagine a grammar that could generate such sequences. If such a grammar exists, then there must be rules R_{ij}, which generate the microstate M_j as an immediate successor of the previous microstate M_i:

$$R_{ij} : M_i \dashrightarrow M_j \quad (8)$$

As the microstates have so far not been coded semantically, it is not possible to find out what the semantic rules are. However, in place of such rules we can determine from our data, by suitable fitting, the conditional probabilities according to which microstate M_i follows from microstate M_j. For six microstates and the corresponding sequential decision structure, the conditional probabilities R_{1i}, R_{2i}, ... for our two populations are shown in Figure 14. From this representation it follows that mathematically highly gifted persons are ore likely to use fewer transitions ("rules") than are normally gifted individuals.

This could shorten the decision-making process and express itself in a reduction in time taken at the action level.

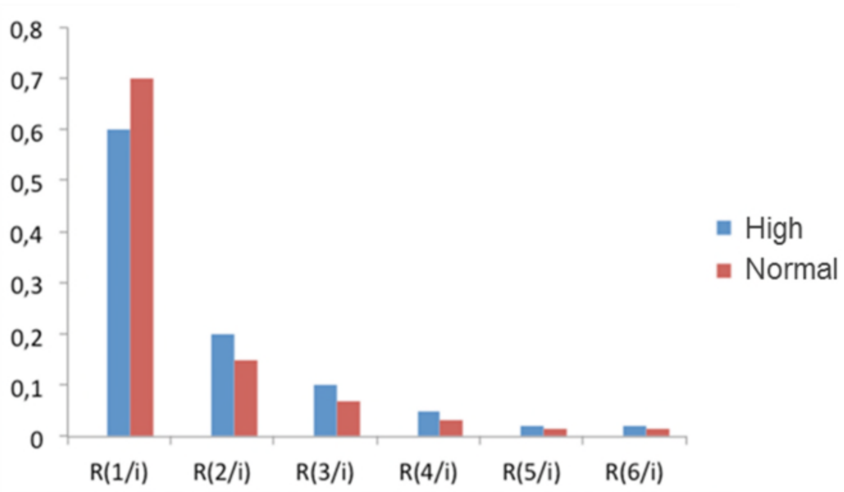


Figure 14. Conditional probabilities R_{1i}, R_{2i}, ... assuming six microstates for mathematically normally (Normal) and highly (High) gifted persons, determined by fitting (i = 1, .., N (set of microstates)).

The ability to find the conditional probabilities of higher order means that valuable rules and their frequencies can be represented. This could point the way toward new possibilities for diagnosis of intelligence.

3.5.3 Subsequences

Seidel (2004) determined the sequences of microstates during the solution of mathematical problems by mathematically normally gifted and highly gifted persons. In Figure 15 the mean solution times and the microstate sequences for one person from each of these groups solving a mathematical problem are shown. The thickness of the lines stands for the frequencies of the transitions. It is conspicuous that the shorter solution times for highly gifted persons seem to correlate with subsequences in the Markov chains. Thus *the time saved by highly gifted persons at the behavioural level can be linked to the shorter sequences of the microstates at the neurological level*. Clearly, highly gifted individuals activate certain concatenated microsequences more often and do not pay attention to others. This could contribute to a shorter solution time. Interpretatively, we can comment, with Klix (1992): Highly gifted people know what the problem's solution depends on and they have solutions at hand.

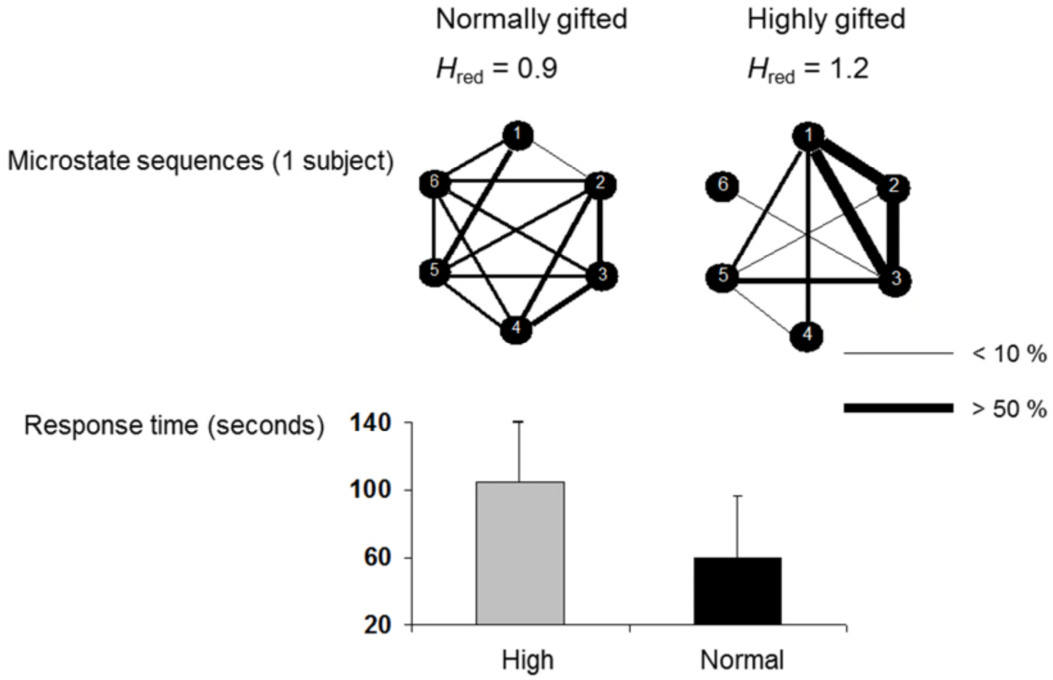


Figure 15. Mean solution times (in seconds) and microstate sequences (Markov chains) for a mathematically normally and a highly gifted individual in solving mathematical problems (Seidel, 2004; Krause, Seidel und Heinrich, 2003).

3.6 A glimpse into the future

The coding of microstates requires systematic analysis. The methods for doing this are already available, for example the difference method of Donders. The same applies to the transformations, in connection with the search for a mental grammar.

The entropy measure must be examined further with regard to the methodological criteria. The spectrum of tasks must be widened to include, e.g., construction, literature, music, sport etc., in order to find out whether this “knowledge that the solution depends on”, a property of highly gifted persons is reflected in entropy decrease irrespective of the kind of problem to be solved.

The computer software must be modernised, so that the assessment times are substantially reduced.

Summary

The processing of information by humans is represented as a Markov chain of microstates. Every cognitive process can be represented as a sequence of microstates.

From the microstate sequences, the entropy decrease is calculated. Mental performance is reflected in the degree of concatenation of microstates. Entropy decrease is proposed as a measure of mental performance. Further testing of this concept is called for.

Sequence analyses at various levels allow conclusions to be drawn about the interactions between these levels. Thus, for example, the shortening of the time required for solving mathematical problems can be related to the formation of specific subsequences of microstates. The microstates can be coded semantically. The long time required for concept activation compared with pattern comparison was found in this investigation to be associated with two additional microstates.

Thus the suspicion is becoming firmer that entropy decrease, based upon more frequent activation of certain concatenations of microstates, may express the creation of order in thought processes.

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