Review on the second edition of the two volumes "Map Projections"
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This book in two volumes is a very interesting combination of a monograph, a text book, and a kind of catalogue of map projections and cartographic map representations of our earth surface on one hand and the mathematical and geodetic theoretical background on the other hand, which can be generalized to the cartography of any celestial object.

The book is very well organized: The mathematical relations are formulated as Theorems followed by proofs and presented in frames, explanations as well as examples are in between and some problems to be solved by the reader make the text interesting also for beginners.

It starts with a differential geometric introduction to the mapping properties between two different Riemannian manifolds, in particular two–dimensional ones immersed into the three–dimensional Euclidian space. Since cartographic maps traditionally represented on two–dimensional plane sheets, the general approach in the first chapter is in Chapter two specialized to the case of mappings from Riemannian manifolds to Euclidian manifolds.

Chapter three is devoted to coordinates on manifolds and their local representations on the charts of the atlantes of the manifolds.

In Chapter four surfaces of Gaussian curvature zero, in particular ruled and developable surfaces are considered since for this class plane representations are particularly straight forward.

Since the surface of the earth is almost a sphere, Chapters five to seven deal with the sphere and mappings of spherical charts to tangential planes, tangential at different points on the sphere. In particular the equidistance, the conformal, the equiareal, the normal perspective mappings an various corresponding projections are presented.

The sphere is only a first approximation of the earth's surface, and the ellipsoid of revolution is a better global approximation which is considered in Chapter eight. With the use of spherical polar coordinates an explicit representation of the ellipsoid and the corresponding Riemann tensor, curvatures and Christoffel symbols are explicitly known. Then the equations of the mapping ellipsoid–of–revolution to tangential planes can be computed, and the equations for the equidistant mapping, conformal and equiareal and the perspective mappings are presented.

In Chapter nine, Gauss' composition of the mapping of the ellipsoid–of–revolution to the sphere with the mapping from sphere to plane is considered to find mapping equations for the ellipsoid–of–revolution as corrections of those for the sphere.

Besides the plane target manifold, simplest ruled surfaces as the cylinder and the cone are considered in Chapters 10 to 19: In Chapters 10 to 13 the mappings sphere to cylinder and 14 to 16 ellipsoid of revolution to cylinder and in Chapters 17 to 18 sphere to cone and in Chapter 19 ellipse of revolution to cone.

For its *world-wide cartographic application*, the chapter 15 on the mapping of the *ellipsoid-of-revolution to the cylinder*, namely the *Gauss-Krueger* or the *Universal Transverse Mercator* (UTM) mapping of the *International Reference Ellipsoid*, is most important that the *Korn-Lichtenstein equations of conformal mapping* subject to an *integrability condition* for UTM are solved. A fundamental solution in terms of a serios expansion subject to the condition of an *equidistant mapping* of a chosen meridian of reference generates the celebrated *Gauss-Krueger conformal mappimg*. The *principal distortion analysis* allows to determine an yet unknown *dilatation parameter*. In a strip $[-3,5,+3,5] \times [80^o S, 84^o N]$ for a strip width of 6^o with 1^o overlap and between $B_S = -80^o$ of Southern and $B_N = +84$ of Northern Latitude, the *dilatation parameter* is fixed to $0,999,578$ or *scale factor* $1:2370$ *(Russian system)*. In contrast, for a strip $[-2,+2^o] \times [80^o S, 84^o N]$ and strip width of 3^o with $0,5^o$ overlap and between $B_S = -80^o$ of Southern and $B_N = +80^o$ Northern latitude the *dilatation* parameter amounts to $0.999,864$ or *scale factor* $1:7353$ (Rest of the World Application), see the detailed *examples* of the *Gauss-Krueger/UTM maps*.

Chapter 20 is devoted to geodetic mappings that map shortest geodets onto straight lines in the plane. Due to Beltrami, such mappings exist if and only if the given surface has constant curvature which is true for the sphere but not for the ellipsoid of revolution. For the latter therefore Riemann coordinates are used which are length preserving with respect to the central point on the tangential plane.

Chapter 21, called Datum problems, deals with our rotating planet within the celestial frame in $\mathbb{R}^3$, the rotating ellipsoid-of-revolution in space; i.e. curvilinear local reference system and the global reference system. Since the Jacobi matrix of the curvilinear datum transformation has extremely bad conditioning, least squares methods and Tykhonov regularisations are used in order to fit the map from local to global coordinates into observed satellite data. The inverse transformation of global conformal into local conformal coordinates is based on the approximation with bivariate polynomials. Stochastic errors are estimated.

In Chapter 22, the harmonic maps as optimal map projections on the sphere are generalized to the ellipsoid-of-revolution. As it turns out, the optimal map for the ellipsoid-of-revolution leads to the vector valued Laplace-Beltrami partial differential equation characterizing the minimal distortion energy. For properly chosen boundary conditions, the Laplace-Beltrami equations then are solved by the use of homogeneous polynomials. The chapter ends with a brief description of variational calculus.

Chapter 23 is an application of the geodetic mappings between particular manifolds as torus, hyperboloid, paraboloid, onion shape, e.g. onto a circular cylinder and mapping of the clothoid for high speed train tracks.

In Chapter 24, the 10 parameter conformal group $C_{10}(3)$ is determined by maximum likelyhood estimates based on the Laplace distribution, the Gauss distribution and the rectangular distribution and three different optimization principles. The numerical results for 15 stations in Germany are presented in different tables.

Appendices 1 and 2 contain explicit calculations for specific conformal transformations and equivariance properties.

The Appendices A–J present mathematical and geodesic fundamental properties and derivations. Special attention is paid to the *ellipsoidal Mercator Projection* as well as to the related *Polycylindrical Projection* in *Appendix I* developed by the authors. Here, *the minimal Airy distortion energy* leads to a *latitude dependent dilatation factor*. It is tabulated for $\pm 2, \pm 4, \pm 6, \ldots$ latitude intervals and has found remarkable application for the territory of *Indonesia* in the *longitudinal range* $[95^o, 145^o]$.

The last pages are filled with 1408 references and a helpful long index.

This book is very valuable for students as well as for the geodesic specialists.

Wolfgang L. Wendland