Das duale wissenschaftliche Paar Moritz-Molodenskij

Geodätische Höhen und Höhensysteme

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1. THEMA
DUALE BEZIEHUNGEN

fest – flüssig
Freund- FEIND
gut – schlecht
SEIN – NICHT SEIN (HEIDEGGER, Martin)
gerade Zahl – ungerade Zahl
rund – eckig
to be or not to be
traurig – fröhlich
ALT – JUNG
alt – neu
KURZ – LANG
Punkt – Strich
weiblich – männlich
endlich – unendlich
NORD – SÜD / OST-WEST
DIESSEITS – JENSEITS

TOT – LEBEN
FRUCHTBAR – UNFRUCHTBAR
BELEBT – UNBELEBT
TAG – NACHT
BERG – TAL
KRIEG – FRIEDEN
VERGANGENHEIT – ZUKUNFT
GEFÜHL – RATIONAL
TIER – MENSCH
NAHBEREICH – FERNBEREICH
ZUFRIEDEN – UNZUFRIEDEN
BERG – TAL
ARM – REICH
GESUND – KRANK
KLAR – UNKLAR
SCHÖN – HÄSSLICH
DICK - DÜNN

1. THEMA
DUALE BEZIEHUNGEN
Mathematik - PHYSIK

plus – minus
positiv – negativ
elektron –positron
Mikrokosmos – Makrokosmos
+ :- -
0 - ∞
Multiplikation – Division
endlich – unendlich
rationale Zahlen – irrationale Zahlen
DISKRET – KONTINUIERLICH
TEILCHEN – ANTITEILCHEN
organisch – anorganisch
TROCKEN – NASS
ORDNUNG – UNORDNUNG
KOMMUNISMUS – MATERIALISMUS
HIMMEL – ERDE
BEKANNT – UNBEKANNT
WELLE – TEILCHEN

MATERIE – ANTIMATERIE
THEORIE – PRAXIS
KONKRET – ABSTRAKT (HENRY CARTAN)
CHAOS – ORDNUNG
POLYCHROM – MONOCHROM
multiplikative – additive
KRUMM – GERADE
LINKS – RECHTS (nicht politisch)
THEORIE – ANTITHEORIE (SYNTHESE)
RAUM – ZEIT
GEOID – TELLUROID
aktiv - passiv
DIV – ROT
RAUMFEST – KÖRPERFEST
Helmholtz Decomposition
KALT – WARM
ROT – BLAU / GRÜN-ROT
SCHWARZ - WEISS

1. THEMA
DUALE BEZIEHUNGEN
MUSIK

einstimmig – mehrstimmig
ANFANG – ENDE
KONTRAPUNKT – PUNKT (Kunst der Fuge)
makro – mikro
E-MUSIK – U-MUSIK
Polyphone – HOMOPHON
DUR – MOLL
schwarze Tasten – weiße Tasten
VOKAL – INSTRUMENTAL
TONAL – ATONAL
HERBST – FRÜHLING
SOMMER – WINTER
GEFÜHL – WACH DENKEN
SCHNELL – LANGSAM
“Minimal Music - ?Maximum Music?”
(PHIL GLASS)

1. THEMA

Einstein Dualität

Mathematik - Realität

As far as the laws of **mathematics**,
they are not certain
and as far they are certain,
they do not refer the **reality**

2. THEMA
div – rot (curl)
HELMHOLTZSCHER ZERLEGUNGSSATZ

Kopplung von

Gravitation - Rotation - Deformation

| | |

Newton - CARTAN - EULER-LAGRANGE

Ansatz

$$\mathbf{V}(x) = \mathbf{V}^L + \mathbf{V}^T$$

orthogonale Zerlegung des Vektorfeldes der Klasse C^2

in: $\mathbf{V}^L - \mathbf{V}^T$: Quelle – Wirbel.

longitudinal oder spheroidal | versus Komponenten | transversal oder toroidal

$\text{div } \mathbf{V}^L \neq 0$

$\nabla \times \mathbf{V}^L = 0$

$\text{rot } \mathbf{V}^L \neq 0$

$\nabla \cdot \mathbf{V}^T = 0$

2. THEMA
div – rot

Beispiel:

laminare versus verwirbelte Strömung

(HYDRODYNAMIK)

ANSATZ: $V(x) = \text{grad } w + \text{rot rot}(e_r v) + \text{rot}(e_r u)$

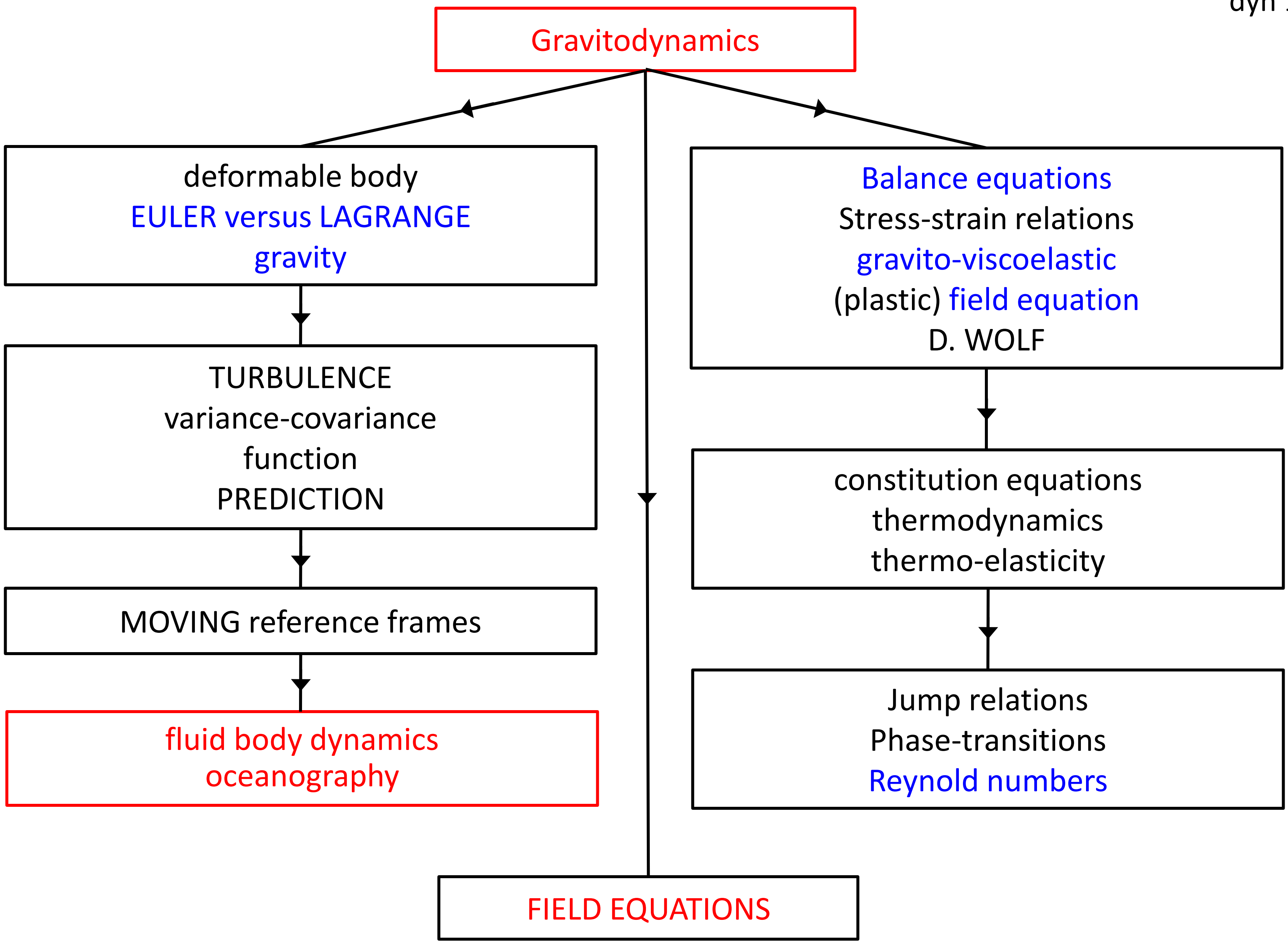
skalare Potentiale $\{u,v,w\}$

BEISPIEL: manuscripta geodaetica **11**(1986) 29-37

„Differential Geometry of the Gravity field“

(E. GRAFAREND)

Planet	Obliquity	Rotational Period	Orbital Revolution	Equatorial Radius [km]	Moons
MERKUR	0.1°	58.65 days	97.97 days	2439	0
VENUS	177.4°	243.0 days	224.7 days	607	0
EARTH	23.4°	0.99 days	365.26 days	6378	1
MARS	25.2°	1.03 days	681.9 days	3393	2
JUPITER	3.1°	0.41 days	11.86 days	71492	63
SATURN	20.7°	0.45 days	29.46 days	60268	60
URANUS	97.9°	0.72 days	83.75 days	25554	27
NEPTUN	29.6°	0.67 days	163.72 days	24769	13


Gravitodynamics
deformable body
EULER versus LAGRANGE
gravity
TURBULENCE
variance-covariance
function
PREDICTION
MOVING reference frames
fluid body dynamics
oceanography
Balance equations
Stress-strain relations
gravito-viscoelastic
(plastic) field equation
D. WOLF
constitution equations
thermodynamics
thermo-elasticity
Jump relations
Phase-transitions
Reynold numbers
FIELD EQUATIONS

FIELD EQUATIONS

curl equ.: $$\mathrm{rot}\,\mathbf{\Gamma} = -2\boldsymbol{\omega}\,\mathrm{div}\,\mathbf{x}^\bullet + \mathbf{\Omega}\,\mathrm{grad}\langle \mathbf{x}^\bullet \mid \boldsymbol{\omega}\rangle - $$
$$- 2\boldsymbol{\omega}\,\mathbf{x}\,\mathrm{rot}\,\mathbf{x}^\bullet - 2\boldsymbol{\omega}^\bullet$$

divergence equ.: $$\mathrm{div}\,\mathbf{\Gamma} = 4\pi g\rho + 2\omega^2 + 2\langle \boldsymbol{\omega} \mid \mathrm{rot}\,\mathbf{x}^\bullet \rangle$$

VORTICITY $\mathrm{rot}\,\mathbf{x}^\bullet$

rigid body

$$\mathrm{rot}\,\Gamma = -2\omega^\bullet$$

$$\mathrm{div}\,\Gamma = 4\pi g\rho + 2\omega^2$$

hydrostatic prepressed elastic body

$$\mathbf{\Sigma} = -p\,\mathbf{I} + c\,\mathbf{E}$$

stress tensor

balance equations

$$\mathrm{d}_{tt}(\rho(\mathbf{x},t)) = \rho\mathbf{I} - \mathrm{grad}\;p =$$
$$= \rho\,\mathrm{grad}\,(\mathrm{U} + \tfrac{1}{2}|\boldsymbol{\omega}\times\mathbf{x}|^2) - \mathrm{grad}\;p$$

↓

CHANDRASEKHAR'S VIRIAL METHOD

virial equations of order zero, one, two and higher order

EQUILIBRIUM figures of higher order

BIFURCATION

↓

3. THEMA
DUALITÄT H. MORITZ – M.S. MOLODENSKij

GEOMETRISCHE GEODÄSIE

GEOMETRISCHE Höhen

(i) Vorschlag von C.F. GAUSS

Projiziere einen toporaphischen Punkt orthogonal auf das REFERENZELLIPSOID

(ii) Höhensystem realisiert von GPS

„GPS Koordinaten“

(iii) TELLUROID

M.S. MOLODENSKIJ

PHYSIKALISCHE GEODÄSIE

Physikalische Höhen

(i) GAUSS – Listing Geoid

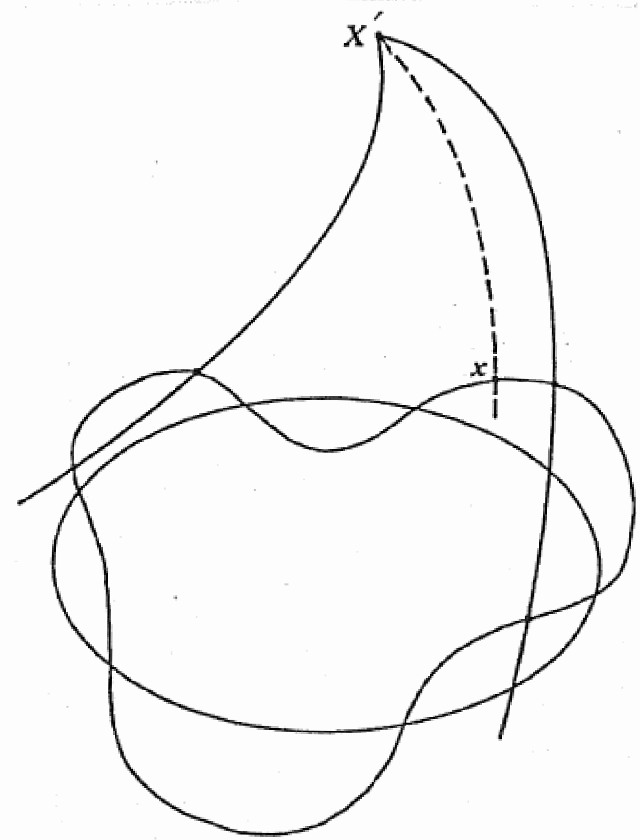
(ii) geometrisches NIVELLEMENT produziert ANHOLONOME KOORDINATEN

(iii) F.R. HELMERT führt die POTENTIAL-Höhe (geopotentielle KOTE) als HOLONOME KOORDINATE ein

(iv) DYNAMISCHE Höhen und Höhensysteme

(v) SOMIGLIANA-PIZZETTI – MORITZ Niveauellipsoid (Rotationsellipsoid, zweiachsig)

(vi) M.S. MOLODENSKIJ entwickelt das Konzept der NORMALHÖHEN basierend auf dem ellipsoidischen SOM-PI-MO Referenzfeld



Projective heights in gravity space, minimal distance mapping with respect to a reference equipotential surface, the *geoid* $w_0 = (62,636,856.5 \pm 3)\ \mathrm{ms}^{-2}$, *at a reference epoch* t_0, *orthometric height* h_G (length of the plumbline / orthogonal trajectory from $X \in T^2$ to $x \in$ GEOID), representation of the *geoid* with respect to an ellipsoid of revolution $E^2_{a,b}$ according to J. ENGELS and E. GRAFAREND (1992a, b)

Molodensky potential telluroid based on a minimum-distance map. Case study: the quasi-geoid of East Germany in the World Geodetic Datum 2000

The telluroid as introduced by M.S. Molodensky (1945, 1948, 1960) may be considered the best analytical representation of the irregular surface of the Earth. Given the placement vector of a point in geometry space, for instance by GPS (global problem solver), and reference gravity potential in gravity space, the telluroid can be uniquely defined by a properly chosen projection. For instance, astronomical longitude/astronomical latitude (spherical coordinates in gravity space) at a topographic point could be defined to coincide with reference longitude/reference latitude (spherical coordinates in reference gravity space) at a telluroid point in order to establish an isoparametric mapping of the

telluroid. Bode and Grafarend (1982) extensively studied such an isoparametric telluroid mapping with respect to a reference gravity potential which is additively decomposed into (1) the zero-order coefficient of a spherical harmonic expansion of the gravitational potential and (2) the centrifugal potential. In particular, they succeeded in identifying the singular points of such a telluroid map. Here we aim at an ellipsoidal telluroid mapping, which is set up as follows.

A. A. Ardalan, E.W. Grafarend, J. Ihde
Journal of Geodesy (2002) 76: 127-138

Best Fit, Som – Pi Field

Analytically we can formulate the above-stated optimisation problem by minimising the constraint Lagrangean

$$\begin{aligned}\mathbf{L}(x_1,x_2,x_3,x_4) :&= \|\mathbf{x}-\mathbf{X}\|^2 + x_4(W_P - w_p)\\ &= [x - X(x_1,x_2,x_3)]^2 + [y - Y(x_1,x_2,x_3)]^2\\ &\quad + [z - Z(x_1,x_2,x_3)]^2\\ &\quad + x_4[W(x_1,x_2,x_3) - w_p]\\ &= \min_{x_1,x_2,x_3,x_4} \end{aligned} \tag{1}$$

$$(\hat{x}_1,\hat{x}_2,\hat{x}_3,\hat{x}_4) = \min_{x_1,x_2,x_3,x_4} \mathbf{L}(x_1,x_2,x_3,x_4) \tag{2}$$

Definition 1. *Somigliana–Pizzetti field as the gravity field of a rotational ellipsoid.*

Explicit form in terms of fundamental geodetic parameters $\{a, b, W_0, \Omega\}$ (according to Grafarend and Ardalan 1999)

$$\begin{aligned} W(\phi,u) &= \left(W_0 - \frac{1}{3}\Omega^2 a^2\right)\frac{\cot^{-1}\left(\frac{u}{\varepsilon}\right)}{\cot^{-1}\left(\frac{b}{\varepsilon}\right)} + \frac{1}{3}\Omega^2(u^2+\varepsilon^2)\\ &\quad + \left\{\frac{1}{3\sqrt{5}}\Omega^2 a^2 \frac{\left(3\frac{u^2}{\varepsilon^2}+1\right)\cot^{-1}\left(\frac{u}{\varepsilon}\right) - 3\frac{u}{\varepsilon}}{\left(3\frac{b^2}{\varepsilon^2}+1\right)\cot^{-1}\left(\frac{b}{\varepsilon}\right) - 3\frac{b}{\varepsilon}}\right.\\ &\quad \left. - \frac{1}{3\sqrt{5}}\Omega^2(u^2+\varepsilon^2)\right\}\frac{\sqrt{5}}{2}(3\sin^2\phi - 1) \end{aligned}$$

Case study: potential quasi-geoid of East Germany

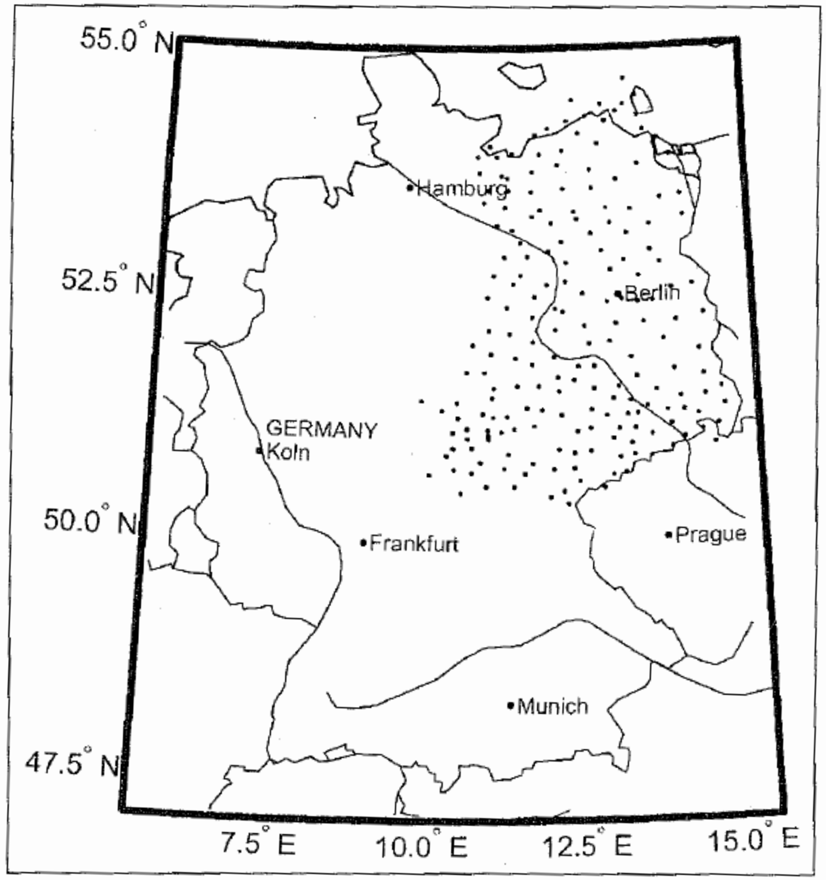


Figure The 196 GPS stations in the eastern part of Germany. Equidistant conic projection; standard parallels: 50°N and 52.5°N; reference ellipsoid: WGD2000

Telluroid Map: East Germany

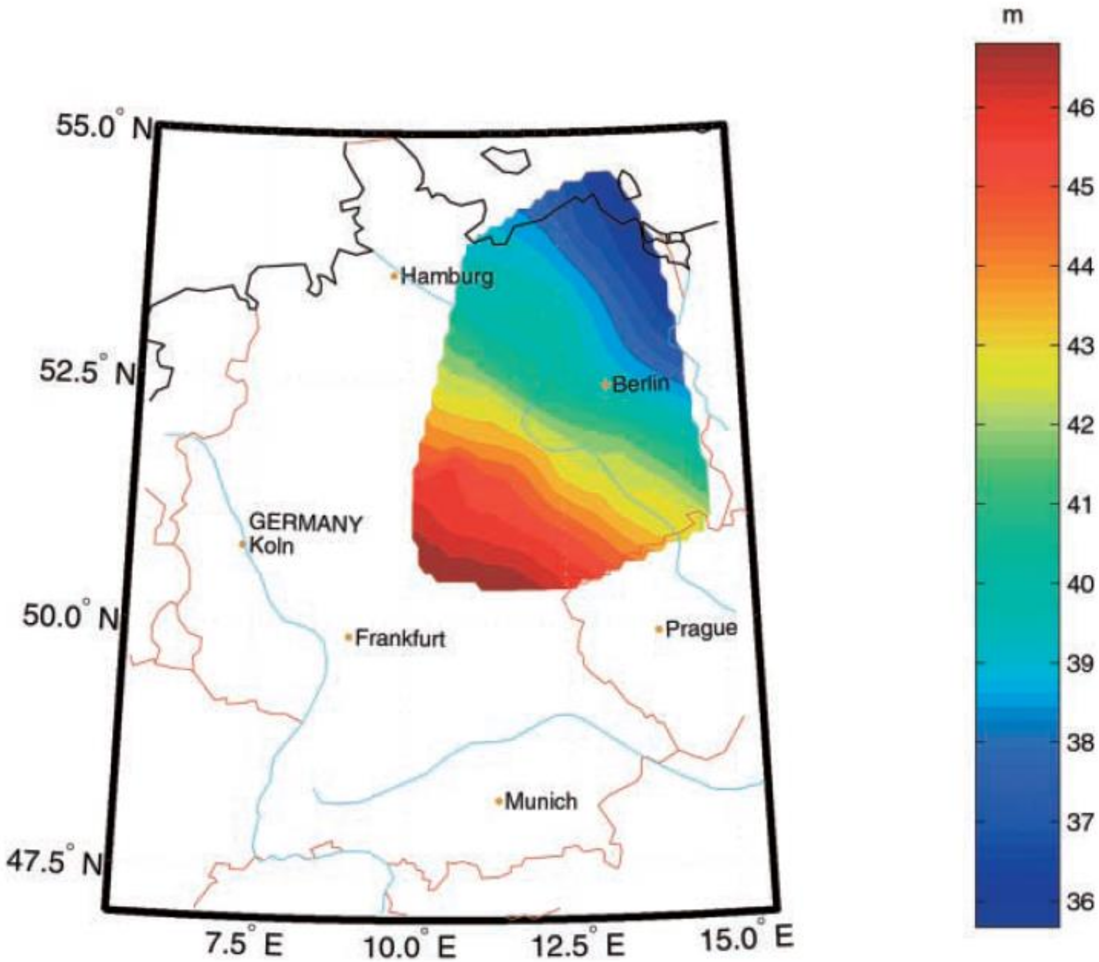


Figure Quasi-geoid map of East Germany, based on the minimum-distance mapping of the physical surface of the Earth to the Somiglian Pizzetti telluroid. The quassi-geoid undulations are in the interval 35.609 to 47.501 m. Equidistant conic projection; standard parallels:50°N and 52.5°N; reference ellipsoid: WGD2000