Sitzungsberichte der Leibniz-Sozietät *78/79(2005), 179–182*

Gottfried Anger

On the Power of a Mathematical Model

The author of this article studied (direct) problems of mathematical analysis in the sense of French mathematicians [1] (Ch.-J. de la Vallée Poussin, M. Brelot, H. Cartan, J. Dieudonne, L. Schwartz, N. Bourbaki, G. Choquet, and so on) from 1952–1970. Then he applied these results to inverse problems in the sense of Russian mathematicians [2], [4] (A. N. Tikhonov, M. M. Lavrentiev, V. A. Morosov, V. G. Romanov, and so on). One can find most results in the author's book on inverse problems [4]. In 1994 he reviewed the two books of Günther Baer in „Zentralblatt für Mathematik und ihre Anwendungen", vol. 844.00002 . This review contains the basic ideas concerning the power of a mathematical model in mathematical physics.

Baer, Günther: Spur eines Jahrhundertirrtums. Neue Gedanken zum physikalischen Weltbild (Track of a giant mistake of a century. New thoughts into the physical worldview). Spur-Verlag, Dresden 1993,
Baer, Günther: Logik eines Jahrhundertirrtums: Neue Gedanken über ‚Kaiser's Kleider' (Logic of a giant mistake of a century). Spur-Verlag, Dresden 1993.

On the Earth's surface all physical (and biological) laws simutaneously hold. As a consequence plants, animals and man have developed during a very long period. For these developments no mathematics is necessary. Scientists assume that the basic physical laws hold in the whole universe.

Experiments can be made only in a laboratory. Such experiments are necessary in order to determine physical parameters. To understand the interactions between the physical forces scientists developed mathematical models which are (sometimes very weak) projections of the real world. It may be that man is unable to understand the reality of nature completely. Therefore practical experience is very important.

If f are the internal parameters of a system (to be determined in a laboratory), the measured data g can be written in the form $Af = g$. In order to determine g we need certain initial and boundary conditions. The relation $Af = g$ is called a direct problem, the determination of the internal parameters $f = A^{-1}g$ (A^{-1} inverse operator) an inverse problem. In most cases of applications A has good mathematical properties (compact operator). If the mapping $A: X \to Y$ is one-to-one on the infinite dimensional spaces X, Y then A^{-1} is discontinuous (F. Riesz 1918). During the past 15-20 years inverse problems have been studied from a systematic point of view. Now we know a little the power of an mathematical model but these results are not well known (see G. Anger, Inverse Problems in Differential Equations, Zentralblatt für Mathematik und ihre Anwendungen, vol. 777.00024).

In the future we have to study in detail the power of a mathematical model from a systematic point of view. This is necessary to understand science and nature. It may be that 90% of the theoretical problems in geophysics and 99% of the problems in astrophysics at large distances cannot be decided by using measured values which are available only on the Earth's surface and near this surface (satellites). The reason is a very large gap of physical information. In order to determine the parameters of a mathematical model by inverse theory (the sources) we need the physical field at all points of the matter. Most results obtained in these fields are mathematical results not of interest in applications.

The solutions of the direct problem $Af = g$ often are representated by integrals. The measurements can be taken only at finitely many points. Therefore the integrals have to be discretized at points y_j. If f is a uniquely solution of $Af = g$, A_k the discretized operator and φ a function which vanishes at y_j then $A_k(f + \varphi) = A_k(f)$. The function $f + \varphi$ is called a ‚ghost'. It may be that for most inverse problems ghosts exist, a very bad situation in physics and medical diagnostics. This fact is almost unknown, but very easy to prove [6], [7], [8].

One can find further remarks on the power of relativity in the homepage of E. Friebe [9], especially the articles of G. O. Mueller and G. Bourbaki, and applications in geodesy in H. Moritz and B. Hofmann-Wellenhof [13].

In mathematical physics the following results hold:

1. Mathematics is the language of physics because, for most physical processes, it allows the formulation of a meaningful description
2. Unfortunately, for the complex systems of real nature, there is no mathematical systems theory: „Science is patchwork".
3. For such systems only certain partial information can be measured, and practical experience is needed to draw at least some meaningful conclusions from the measured data: "praxis cum theoria".

4. Engineering systems are relatively well known and managable. For them, a mathematical systems theory usually exists.
5. Biological systems are much more difficult: measurements may not be reliable and meaningful, and mathematical systems theories hardly exist. Thus in medicine practical experience (“ars medica”) is particularly important.
6. Social sciences are between: systems theories do exist to a limited extent, and computers can be used with a certain success.

In relativity the power of the corresponding mathematical models were not studied from a systematic point of view. Using the results of G. Green (1828) [11] the scientists P. Pizzetti and G. Lauricella proved in 1907-1912 that the mass distribution of the Newtonian potential cannot be uniquely determined by its values on the boundary of a domain ([4], [11], [12]). Up to 1980 no scientist was interested in these results, which has basic consequences for the applications of mathematical physics [12], natural sciences and medicine. Similar results hold for all mathematical problems relative to the universe. The following main problem is almost unknown in mathematical physics [4]: *Which external parameters of a system are to be measured on which subsets of the R^n in order that certain coefficients or the source term or functionals of these coefficients or the source term inaccessible to measurements can be determined in a stable and unique manner?* Similar problems have to be studied in relativity (and the universe).

The level of theoretical physics largely depends on the level of mathematics.

References

[1] G. Anger (1958), *Sur le rôle des potentiels continues dans les fondements de la théorie du potentiel.* Séminaire de Théorie du Potentiel, tome 2 (1958), 30 pp., Institut H. Poincaré, Université de Paris.

[2] G. Anger (Hrsg.), *Inverse and Improperly Posed Problems in Differential Equations*, Conference held at the Martin Luther University Halle-Wittenberg, May 29-June2, 1979. Mathematical Research, vol. 1. Akademie-Verlag, Berlin 1979.

[3] G. Anger (1980), *Lectures on potential theory and inverse problems.* In: Die moderne Potentialtheorie als Grundlage des inversen Problems der Geophysik, 15–95. (Sommerschule der KAPG Arbeitsgruppe 5.2, Bergakademie Freiberg (Sachsen), 5.-16. Juni 1978), Geodätische und Geophysikalische Veröffentlichungen, Reihe III, Heft 45 (1980).

[4] G. Anger (1990), *Inverse Problems in Differential Equations*. Plenum Press. London 1990.

[5] G. Anger, R. Gorenflo, H. Jochmann, H. Moritz and W. Webers (eds.) (1993), *Inverse Problems: Principles and Applications in Geophysics, Technology and Medicine.* Conference held at the GeoResearchCenter Potsdam, August 30–September 3, 1993. VCH Weinheim, (reprinted 1998).

[6] G. Anger (2004), Homepage, http://members.aol.com/GottfriedAnger/myhomepage, 254 pp.

[7] G. Anger and H. Moritz (2004), *Inverse Problems and Stability – Basic Ideas and Applications.* In Homepage. www.inas.tugraz.at/forschung/InverseProblems/AngerMoritz.html.

[8] G. Anger (2005), *Die Komplexität der Natur und ihre Interpretation.* In: Was von moderner Physik bleibt und fällt. Verlag Kritische Wissenschaft, D-51556 Windeck.

[9] E. Friebe (2004), *Relativitätstheorie, Diskussion mit G. O. Mueller,* in www.ekkehard-friebe.de, see also Georges Bourbaki in www.ekkehard-friebe.de.

[10] J. Grolle (2005), *Ritt auf dem Lichtstrahl – Einsteins Weg zur Relativitätstheorie.* Spiegel 3, 134–140.

[11] O. D. Kellogg (1929), *Foundations of Potential Theory.* Springer, Berlin.

[12] A. Marussi (1980), *On the density distribution in bodies of assigned outer Newtonian attraction.* Boll. Geof. Teor. Appl. 22, 83–94.

[13] H. Moritz and B. Hofmann-Wellenhof (1993), *Geometry, Relativity, Geodesy*. Wichmann, Karlsruhe.

[14] G. O. Mueller (2004), *95 Jahre Kritik der speziellen Relativitätstheorie,* in www.ekkehard-friebe.de.

[15] W. Neundorf, Die Physik in der Sackgasse? Wissenschaft und Kritik. In: www.neundorf.de.