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Eratosthenes and the heliocentric hypothesis of Aristarchos

Abstract

At the end of his treatise "Psammites" (Sandreckoner) Archimedes pointed out the decisive role of the distances and sizes of the celestial bodies regarding the heliocentric hypothesis of Aristarchos of Samos.

At that time Eratosthenes of Cyrene had measured already not only the circumference of the earth to be $U = 252\ 000$ stadia $= 40\ 000$ km, but also the "Astronomical Unit" to be AU $= 804\ 000\ 000$ stadia $= (10\ 000 + 50)$ earth diameter $\sim 130\ 000\ 000$ km.

The heliocentric methodology of antiquity can be reconstructed since it was strictly based on geometric concepts using eccenter/epicycles, well-known to the ancients. And indeed, the distances between the celestial bodies play a decisive role in this geometrically based methodology as well as the cinematics of eccenter and epicycle.

This first serious methodology of celestial mechanics was, according to Ptolemy, used by all the other astronomers at the time of Hipparch. And indeed, a serious geocentric methodology as later on used for about 15 centuries was developed not before Ptolemy, as he stated explicitly in the Almagest.

1 The role of distances in ancient astronomy and geodetic methods

It was already Platon who had pointed out very urgently that stereometry must be studied before astronomy, that means, the distances between the celestial bodies must be taken into account, in addition to what Eudoxos of Cnidos did using his homocentric spheres.

When Archimedes of Syracuse described in his treatise "Psammites" (San-

dreckoner) the heliocentric hypothesis of Aristarchos (as well as some conclusions about the cosmos coming out from that) for Gelon II, son of his king Hieron II, he finished his treatise with the following remark.

I believe, king Gelon, that all this will appear incredible to the many people who have no part in the mathematical sciences.
But not at all to the educated one who have reflected upon the

- ***distances and sizes***

of the earth, the sun, the moon and the whole universe.

The famous mathematician Archimedes was later on honored by Carl Friedrich Gauß (together only with Isaac Newton) with the title "Illustrissimus". And certainly he considered himself as an educated man. Was he, then, an "Adherent of Aristarchos", when he made his famous utterance: "Give me a place where I can stand and I will move the earth."?

In any case the distances and sizes of the celestial bodies played obviously a deciding role in the cosmic considerations of the Greek scientists at the time of Archimedes. If somebody does not take notice of this fact he will easily get a wry conception of the development of astronomy in antiquity.

But did the scientists at that time only reflected upon or did they measure already the distances and sizes of the celestial bodies? Indeed, at least one of them did it very carefully: Eratosthenes of Cyrene.

By ancient literary information in combination with modern knowledge about the metric length of foot-units employed by the ancients it can be shown that Eratosthenes (as before him e.g. Heracleitos) used a stadion unit defined by

- 1 stadion $= 600$ Gudea-foot $= 600 \cdot 0,26455$ m $= 158,73$ m.

He determined therefore already in the 3. century BC surprisingly precise the

- circumference of the earth $= 252\,000$ stadia $= 40\,000$ km.

Moreover, there are several remarks in the ancient literature (Pseudo-Plutarch, Stobaios, Pseudo-Galenos, Theodoretos, Eusebios of Caesarea, Johannes Lydus, Ptolemy-Scholiast), that he also provided a value for the distance earth/sun, in modern times called "Astronomical Unit".

After a careful analysis of those literary texts and a correction of a very few simple and plausible scribal errors by Andreas Kleineberg it turned out that it was always the same value, namely,

- AU $= 804\,000\,000$ stadia $= (10\,000{+}50)$ earth diameter $\sim 130\,000\,000$ km.

Using the well-known angular diameter of the sun he could get then easily the

- diameter of the sun $= 100 \cdot$ diameter of the earth.

Regarding the huge size of the sun according to those data and the tiny size of the earth there can be few doubts that Eratosthenes was "Beta", the second "Adherent of Aristarchos". When he was an adherent he could have tried to determine the topocentric parallaxe of Venus in conjunction (when the Venus was nearest the earth) and from this he could easily derive the topocentric parallaxe of the sun and from this again his astonishing precise value for the distance earth/sun.

As long as we have no contradictory information we may assume (at least as a working hypothesis) that the heliocentric hypothesis was fully elaborated to a heliocentric methodology for practical applications by the four Alexandrian "mathematici" Aristarchos of Samos ($\sim$ 310–230), Archimedes of Syracuse ($\sim$ 285–212), Eratosthenes of Cyrene ($\sim$ 284/274–202/194), Apollonius of Perge ($\sim$ 262–190). And also that the distances of the celestial bodies played a deciding role in this elaboration as well as the cinematics of eccenter and epicycles.

Ptolemy must have known about such a heliocentric methodology when he remarked in the Almagest, book I.7, about the heliocentric hypothesis and the "Adherents of Aristarchos":

Regarding the phenomena in the starry sky, even if perhaps by the ***greater simplicity*** *of this* (heliocentric) *concept no hindrance would be that this is so, it escaped those men, that due to the properties adhering to ourselves and the peculiar atmospheric circumstances the ridiculousness of such an assumption must become obvious.*

Of course, distances to and between the celestial bodies could not be measured directly. They could only have been obtained by a connection of the astronomic angle measurements with geodetic distance measurements.

In this respect three principal methods could have been of help:

1. topocentric parallaxes (1° for the moon, $<1'$ for Venus and sun) or forward incision (Thales; fixed stars), respectively,

2. backward incision (used by Ptolemy for the outer planets),

3. Keplerian ellipses constructed using an epicycle (equant model).

In any case, regarding distance determinations in ancient astronomy a combination of geodetic and astronomic methods was necessary. It seems therefore to be in particular a task of the geodesists to reconstruct those methods.

2 On the methods of the "other" astronomers at the time of Hipparch

Neugebauer (1975, S.277) remarks about Hipparch.

For us the main problem consists in the clarification of the position of Hipparchus with respect to his predecessors and followers. He himself names Archimedes as preceding him in the investigation of the length of the year. We know that Apollonius reached ***full mastery*** *of the cinematics of eccenter and epicycle. We also know how much Ptolemy contributed to the lunar and planetary theory. And we are aware of important data utilized by Hipparchus which originated in Mesopotamia*(Seleukos of Seleukia). *Obviously only by a*

- ***careful analysis of technical details***

may one hope to obtain a valid picture of the astronomy of Hipparchus and his time. To call him the ***"father of astronomy"*** *does not solve the problem.*

With respect to planetary theories Neugebauer (1975, S.342) added later.

We know practically nothing about Hipparchus' planetary theory excepting what we can conclude from few introductory remarks by Ptolemy in the Almagest.

Indeed, Ptolemy remarked there (book IX.2):

He (Hipparchus) *had not even laid down the basics for a theory of the five planets in the commentaries delivered to us. He has only ordered the observations of those for a more beneficial employment.*

It is absolutely necessary for an understanding of the development of ancient astronomy to take notice of the fact, that a geocentric methodology was developed for the first time by Ptolemy; before Ptolemy, there was no adequate geocentric methodology at all. Ptolemy continues:

It seems that he (Hipparch) *thought to give not merely the declaration*

- *that any planet shows a* ***double anomaly*** *and*

- *that each shows* ***unequal retrograde distances***

whereas the other astronomers (at his time) *conducted their proofs*

- *on the way of* ***geometric constructions***

- *under the assumption of* ***one and the same anomaly as well as retrograde distances***.

Two comments may be opportune here. First, in geometric constructions the distances will of course always play a decisive role. Second, one and the same retrograde distance takes place then and only then, if a planet would move in a **concentric circle** around the sun. Ptolemy continues:

Likewise he (Hipparch) *restricted himself not only to the declaration that those phenomena may be expressed by*

- *the assumption of eccenter*

- *or circles, concentric to the ecliptic, which put epicycles into rotation*

- *or even by a combination of both types of circles*

- *where comes out such and such great the anomaly related to the sun,*

- *such and such great the anomaly related to the ecliptic* (fixed stars).

For that has been used by nearly all who wanted to indicate by means of so-called "Tablets for eternal ages" the uniform motion on circles, but they did it completely wrong and remained indebted for the proof $\cdots$.

Indeed, a definite proof was given only much later by mensuration, namely in 1838 AD by Friedrich Willhelm Bessel when he observed for the first time the extremely small parallaxe of a fixed star.

A main problem for us is not only the clarification of the position of Hipparchus with respect to his predecessors and followers, but in particular his position with respect to his contemporaries such as Seleukos of Seleukia.

Fortunately, the short remarks given by Ptolemy are sufficient for this task due to the fact that the "other" astronomers at Hipparchus' time based their models strictly on **geometric constructs** using **two anomalies** as well as the **cinematics of eccenter and epicycles**.

3 Geometric constructs of the cinematics of eccenter and epicycles

Aiming for "a careful analysis of technical details", as suggested by Otto Neugebauer, we may be led by his remark in (1975, p.698).

Without the

- ***accumulation of a vast store of empirical data***

and without

- ***a serious methodology for their analysis***

the idea of heliocentricity was only a useless play on words.

3.1 Anomaly related to the sun: Rotating epicycle and excenter

One of the most conspicuous celestial phenomena regarding the planets are the periodically changing brightness of Luzifer, as the Venus was called, too, in ancient times as well as of Mars and Jupiter in conjunction and opposition. Those could not be explained by the rotating sphere model of Eudoxos, but the distances must be taken into account.

Excenter/epicycle models, probably developed already by the Pythagoreans, could be useful in this respect. Considering retrogradation phenomena Ptolemy introduced **rotating** excenter and epicycles in the Almagest in book XII.1 with the following remark.

Before they approach an investigation of this phenomena, the other mathematici (who?) *and also Apollonius of Perga mention before a theorem which sounds different*

- ***whether the anomaly to the sun, which are considered to prevail alone with this***

be expressed according to the epicyclic or eccentric hypothesis. The reader may remember that the length of the retrogradation distances of the outer planets do not depend on the anomaly related to the sun but only on the anomaly related to the ecliptic.

On the other hand, using the basis geometric construct of the "other" astronomers, namely concentric circles around the sun, the major question already Aristarchos must have faced will be: How do the orbits of the outer planets look like for an observer at the earth?

And for this question the rotating eccenter model of Apollonius of Perge and the other mathematici provides a simple and evident, strict **geometric** answer as can easily be seen from fig. 1(b).

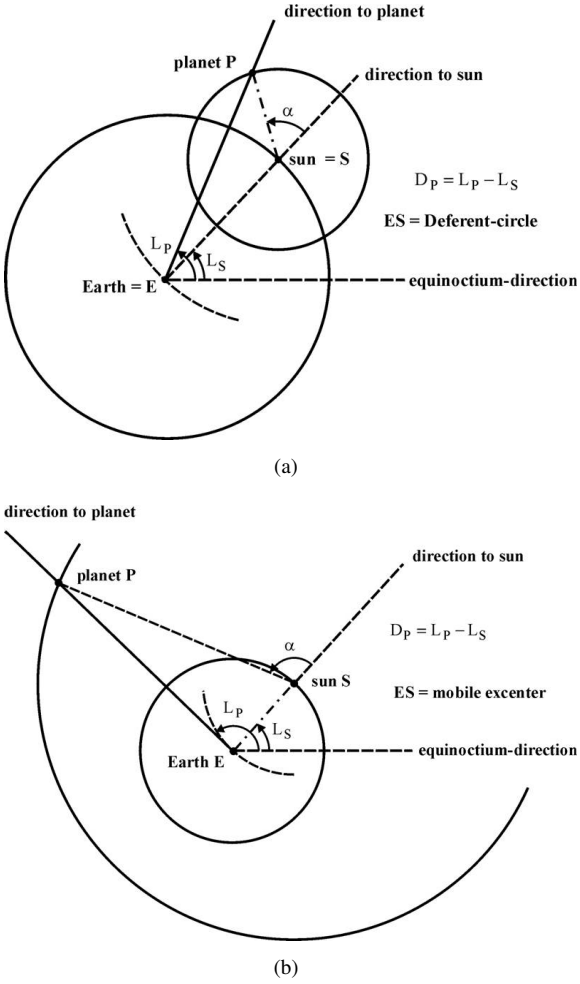

Figure 1: Rotating epicycle (a) and eccenter (b)

3.2 Anomaly related to the ecliptic: static eccenter and equant model

3.2.1 Static eccenter and static epicycle

Already Aristarchos of Samos must have known, that the motion of the planets cannot be modelled by concentric circles around the sun, because of phenomena strictly depending on distances and on the ecliptical longitude:

1. Different length of the seasons, as already carefully and precisely determined by Callippos at around 330 BC, according to their ecliptical longitude.

2. Different length of the retrogradation distances of an outer planet according to its ecliptical longitude.

3. Different maximal elongations of an inner planet according to its ecliptical longitude.

4. Different angular diameters of the moon according to its angle (true anomaly) to the perigeum of its orbit.

The most simple idea would have been to introduce eccentric orbits instead of concentric orbits around the sun. This can be done most simply either by a static eccenter (as used by Ptolemy for the sun) or by a static epicycle, respectively (see fig. 2)

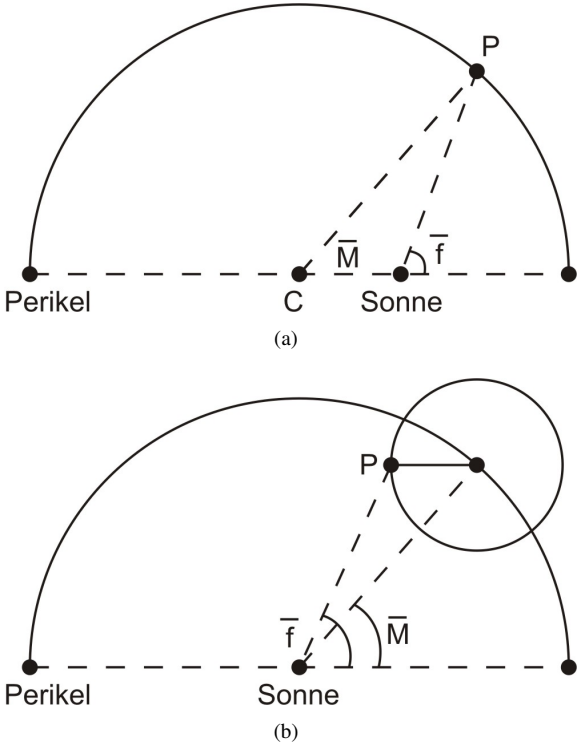

Figure 2: Static eccenter (a) and epicycle (b)

3.2.2 Insufficiency of the static eccenter/epicycle model

Certainly, the static eccenter/epicycle construct is not too bad to "save the celestial phenomena". Only if very precise observations of the planetary positions are performed, then and only then it must turn out that it is not sufficient to connect those by such an eccenter kind of geometric model.

Do we have any information about such kind of precise observations? Indeed, Ptolemy reported in the Almagest, book IX.2:

As for the establishment of the anomalies no small difficulty brings along the perception that at any planet two apparent anomalies do show an influence on them; those are unequal, regarding both their seize and their recurrence time. One of them will be set up in ***causal*** *relation to the sun, the other*

one in relation to the parts of the ecliptic. However, both are throughly mingled together; so consequently it is difficult to eliminate the peculiarity both adhering individually.

He proceeds then, regarding now the old observations:

A further inconvenience is that the records of the old observations are to a large extend kept very unappreciative and superficial.
The more coherent ***series of observations*** *are concerned with*

- *standstills and*
- *heliacal risings and settings.*

Indeed, those are just the kind of observations necessary to check whether a static eccenter/epicycle model will be adequate to connect the individual precise astronomic observations of the planets (see fig.3).

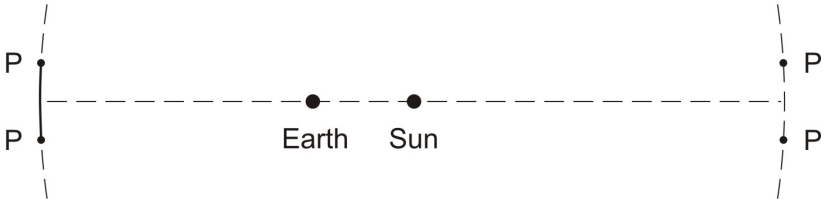

Figure 3: Standstills and heliacal risings and settings of the outer planets

The result of precise observations of stillstands and heliacal risings and settings must have been that the static eccenter/epicycle model must slightly be modified to "save the celestial phenomena".

Taking into account heliacal risings and settings another result must have been that all planets (with perhaps the exception of the earth) move in nearly circular orbits around the sun, as found out later on by Tycho Brahe. But Ptolemy avoids to use heliacal rising and setting data within his model of planetary motions. Willfully or unconscious, that is here the question.

3.2.3 A slight modification of the static eccenter model: Keplerian ellipses

One may modify the static eccenter model by a small epicycle (see fig. 4). When the center of the epicycle moves around an angle z in the retrograde

direction, the planet P on the epicycle may move in the prograde direction about the same angle z. The orbit of the planet constitutes then a Keplerian ellipse.

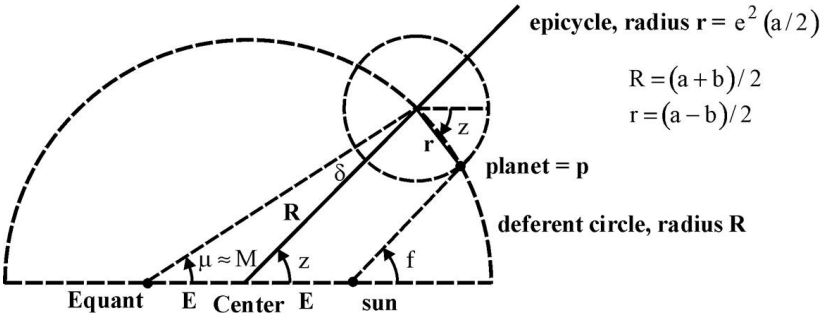

Figure 4: Keplerian ellipse constructed with ruler and circle

The reader can easily check by himself that

$a = R + r =$ semi major axis

$b = R - r =$ semi minor axis

$z =$ eccentric anomaly

$f =$ true anomaly

$\mu \sim M =$ mean anomaly

Apollonius of Perge was and is very famous for his treatise "Conics". Can it be that he never detected how to draw an ellipse by ruler and circle? Was it perhaps explained in the lost book of the "Conics"?

3.2.4 A clever approximation of a Keplerian ellipse: Equant model

Regarding the ancient accuracy of astronomic observations a clever approximation of the elliptic orbit is near at hand. If the orbit excentricity e = E/a is small, as it is the case for the moon and the planets, one gets

$$(r/a) = (e^2 + (e^2/2)^2 + \cdots)/2 \sim 0.$$

The result is the so-called equant model (see fig. 5(a)).

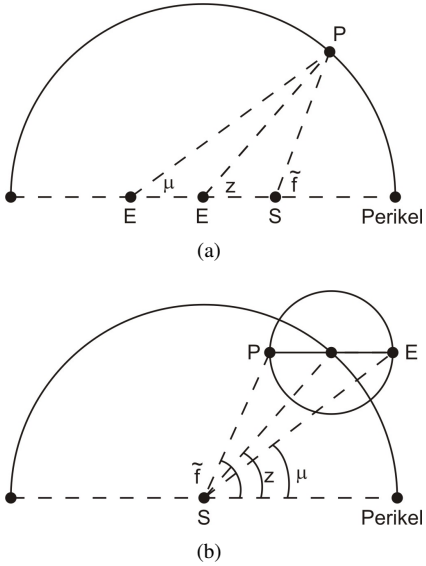

Figure 5: Eccentric (a) and epicyclic (b) equant construct

Of course, the equant model can be used, such as the mobile as well as static eccenter/epicycle model, in an eccentric form (fig. 5(a)) as well as in an epicyclic form (fig. 5(b)). The epicyclic form was used by Ptolemy in the second part of his lunar theory.

In the frame of a uniform heliocentric methodology the orbits of all the planets including the earth must be modelled by an equant model.

Regarding the ancient observation accuracy, however, it would have been geometrically completely sufficient in case of the outer planets to approximate the earth orbit by a concentric circle around the sun. Also the Venus orbit may be approximated by a concentric circle. This is what Ptolemy did in the Almagest.

In case of the Mercury, however, both his orbit as well as the earth orbit will require a modeling by the equant concept as will be necessary, too, if very precise observations of Mars are at present such as performed by Tycho Brahe.

4 A comment on Ptolemy's lunar theory

Neugebauer (1975, p.86) remarked.
It is interesting, to find here in the lunar theory the roots of the concept of the "equant" (-model) *which plays such an important role in Ptolemy's planetary theory, that is, the idea that uniformity of motion may refer to a point different from the geometric center of a circular orbit.*

The motion of the moon itself on the epicycle requires a special investigation which has its parallels in planetary theory. It hardly can be doubted that it was the theory of the lunar motion where the modifications of the simple epicyclic motion originated.

Indeed the epicycle in Ptolemy's lunar model can be considered just as an epicyclic equant construct. Ptolemy designated book IV and V of the Almagest to the moon. In chapter IV.1-8 he started his treatment with the so-called first anomaly. In chapter IV.9 he started his investigations again using other, "more agreeable" methods after he has remarked as an introduction.

By application of more agreeable methods, independent of the previously made presuppositions to get the aspired results, we have recently found as incorrect those positions in (ecliptical) *latitude which have been calculated with the help of the former presuppositions.*

According to the now independently of that located positions we have set right those hypotheses which are concerned with ***the sizes and distances****, after we have proofed their unsteadiness.*

We have used the appropriate method with the hypotheses of ***Saturn and Mercury*** *under removal of some former outcomes, which have not been obtained sufficiently accurate, since later on better founded observations have been handed down to us.*

It is revealing that Ptolemy connects in this passage his **advanced** lunar theory with the hypotheses of Saturn and Mercury. For Saturn he used the equant model. His model for Mercury can be considered as a − not correct one − approach to model both the orbit of Mercury and of the earth by an equant model.

Moreover, his advanced model for the first anomaly of the moon, which he developed in the second part of book IV, can easily be explained as an equant model of the epicycle type. Ptolemy proceeds further.

For who approaches with true veracity and untiring thoroughness the theoretical treatment of those circumstances he should not alone utilize those

means and ways offered by modern times

- *for a rectification of the old hypotheses*

but also

- *for the rectification of his own hypotheses*

if they will need improvement.

Considering the greatness and divinity of the system of instruction and feeling himself competent to proclaim it, he should not keep it as a disgrace, if to him a rectification be allotted from another side and not only from its own perception.

There can be few doubts, according to this profession, that Ptolemy found other reports on which he based his advanced method. Unfortunately, he did not tell us who was the author of such reports and what were the contents of those.

In any case the modern myth that Ptolemy's lunar theory is **not** based on the (epicyclic) equant model can easily be doubted and would certainly require additional plausible arguments.

In his book "The crime of Claudius Ptolemy" R.R. Newton (1977, p.208) comes to the conclusion.

In fact, the Greek knowledge of the apparent size of the moon was compatible with an elliptical orbit but not with an epicycle/eccentric model of motion. It was compatible (of course) *with the equant model. But this model was never applied to the moon, although it was applied to the planets.*

As be said, his last remark remains very doubtfully.

5 An orbiting earth and the parallaxes of the fixed stars

If the earth moved, according to Eratosthenes, in a circle with a diameter of about $(20\,000+100)$ earth diameter around the sun, then in principle it should be possible to observe a half-yearly parallaxe of a fixed star and derive from this its distance. Since no parallaxes could be observed till 1838 AD all fixed stars must be very far away. Since the light of those objects reached the earth, they must be very huge celestial objects like the sun.

Of interest in this respect are the following remarks in (Neugebauer 1975, p.584 an p.611).

*Interestingly **Geminus** denies the existence of a single sphere for the fixed stars, which he assumes to be distributed in depth. The same opinion is also held by **Proclus.** In modern times **Seleucus** has become famous as supporting Aristarchus in the assumption of an axial rotation of the earth, relating it to his theory of tides, and by his assertion that **the cosmos is infinite.***
No doubt, Geminus, Proclus and Seleucus had already a correct conception about the structure of the universe.

6 Concluding remarks

From the point of view of mensuration there is nothing mystical, there is no "Greek spirit" left about the developement of Greek mathematics and the very old natural sciences geodesy and astronomy.

It started when Thales/Anaximander, probably for the first time, combined angle and length measurements for very practical purposes such as navigation and geography. It ended when geodetic methods have been used to determine the distances between the celestial objects.

It was just the investigation of the distances of the celestial bodies which must have led to the heliocentric hypotheses as well as its elaboration to a heliocentric methodology. To understand it, we have first to reach for ourselves full mastery of the cinematics of eccenter and epicycle such as Apollonios of Perge had got it already; this is provided in the present paper in section 3.

The present treatise is merely based on "a careful analysis of technical details" as required by Otto Neugebauer. It comes to the same conclusion as Bartel Leendert van der Waerden (1988), namely, that the heliocentric hypothesis of Aristarchos played a much greater role in antiquity as usually assumed in modern times. Indeed, according to Ptolemy all the other astronomers at the time of Hipparch have used a heliocentric methodology.

All the questions and riddles turning up until today with the Almagest will vanish, as far as we could see, after the reconstruction of the heliocentric methodology, probably developed by the "mathematici" Aristarchos, Eratosthenes, Archimedes and Apollonius. A very rational image of the development of geodesy and astronomy by the Greeks turned up then, too; no contradiction could be found yet by us with ancient literary information (Lelgemann 2010).

The movement of the planets have been of interest, of course, not for the common men but only for philosophers and astrologists. It remains to anal-

yse the numerical data in the many horoscopes in particular before 150 AD, whether they agree or do not agree with such kind of heliocentric methodology as reconstructed in this treatise.

Why the geocentric and not the heliocentric concept survived for about 15 centuries is an iteresting question. Ptolomy's concept with the physical hypothesis of Aristoteles. If somebody doubted Ptolomy's heliocentric concept, he first had to critisize the physical hypothesis of Aristoteles. This was done in a very voluminous treatise by Wilhelm von Ockham at about 1320 AD. Short time later the heliocentric concept was established again stepwise by Nicolaus of Oresme, Nicolaus of Kues, Nikolaus Kopernigk, Johannes Kepler, and finally by Isaac Newton. The mesurments of Friedrich Wilhelm Bessel removed the last doubts obout its correctness.

Acknowledgment

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