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Epicycles in Modern Physics

Section 1. Introduction

Being perfect celestial bodies, the planets must rotate in circular orbits around the Earth as the center of the Universe. This, at least, was what the Greeks' idea of perfection taught them. In reality, however, planetary motion was much more irregular. In the 2nd century AD, Ptolemy explained the complicated motion of the planets, as seen from the Earth, by their moving on small circles about a point lying and moving on the central circle about the Earth. As observations became more and more accurate, more and more of such epicyles were superimposed, so that the whole system became very complicated. This may seem to us ridiculous and childish, but mathematically it is nothing else than an expansion of the orbit into trigonometric series used even today ... to which we shall refer later as

Trick 1: the mathematical trick

Thus mathematically, epicycles were perfectly workable, but our intuitive sense of perfection or elegance of nature was not satisfied. Therefore, two additional ideas were introduced:

Trick 2: the Copernican trick, in more distinguished terms known as the Copernican Revolution (*around 1500*), simply shifting the perspective, placing the Sun rather than the Earth at the center of the universe. This considerably changed man's philosophical concept of the universe. However, practical astronomy and geodesy, for good reasons, blandly use geocentric reference systems as well as heliocentric systems, whichever is more convenient. And Einstein's General Theory of Relativity declares that both reference systems are equivalent, for good reasons!

But let us not forget that, also for good reasons, in those times, people were ready (or not ready) to die for their convictions, and Giordano Bruno did die on the stake. Therefore, we should not be too arrogant about our modern „superiority"!

Trick 3: the Keplerian trick. The Copernican viewpoint removed many of the apparent anomalies, but not all. The anomalies vanished, however, almost completely if the circle was replaced by an ellipse. This is what Johannes Kepler did *around 1600* (only to give you round numbers). We may say: what is the difference, an ellipse is almost as simple and elegant as a circle, but for renaissance-trained minds it was a great step ahead.

The next step is well known. Newton (*around 1700*) deduced the principles of *classical mechanics* rigorously from Kepler's laws of planetary motion. Conversely, we may use Newton's laws of motion and of gravitation to get back Kepler's elliptical orbits....but with small corrections due to the mutual attraction of the planets! (This is a nice dialectic step, similar to children emancipating themselves from their parents.) Fortunately, as Alfred North Whitehead wryly remarked, the accuracy of Kepler's, or rather Tycho Brahe's, measurements was not too great, otherwise Kepler would never have discovered his nice ellipses, and Newton would not have derived his fine laws.)

Around 1800, Newton's classical mechanics reigned supreme as final physical theory, as the following quotation from Laplace shows:

„An intelligent being which, for some given moment of time, knew all the forces by which nature is driven, and the relative position of the objects by which it is composed (provided the being's intelligence were so vast as to be able to analyze accurately all the data), would be able to comprise, in a single formula, the movements of the largest bodies in the universe and those of the lightest atom: nothing would be uncertain to it, and both the future and the past would be present to its eyes."

The concepts of epicycles and the three basic tricks will be useful for our further considerations.

Section 2. Relativity

Towards 1900, Newton's classical mechanic reigned supreme, like the Greeks' great circle, surrounded as „epicycles" by supposedly minor phenomena such as Faraday's and Maxwell's electromagnetism and Planck's hardly understood quanta. Another change of paradigm in the sense of Kuhn [1] appeared to be *due around 1900*.

In 1905, Einstein unified all these ideas into the two great theories of the 20^{th} century: Relativity and Quantum Theory.(In this section we only speak about relativity.) In fact, the situation was a little more complicated. Einstein discovered the *Special Theory of Relativity* proclaiming the equivalence of

„inertial“ reference systems, which was already known by Gallileo Galilei, using, instead of the Galilei transformation, *the Lorenz transformation* which was also already known (e. g., by Poincare for Maxwell's equation).

So what was the great discovery of Einstein? He recognized that the Lorentz transformation was really valid *generally* for both electromagnetism and mechanics. This idea alone was sufficient to make him one of the intellectual giants of all times. That is the idea of *unification*, we also recognize the „Copernican trick“ of changing the perspective rather than the formulas.

The next step was made two years later by the mathematician Hermann Minkowski, who recognized that Einstein's theory implied the idea of a four-dimensional (pseudo-)Euclidean unified space-time. Another unification!

Now, having the advantage of hindsight, the next step is logical: a few years later (1915, in the meantime he had studied Riemannian geometry) Einstein recognized that by making Minkowsky's spacetime *curved*, this curvature can be interpreted as gravitation. This meant a third unification, the unification of Special Relativity with gravitation to get the *General Theory of Relativity*. (The fact that in the same year the famous mathematian Hilbert discovered a very similar variational principle, increases the greatness of both Einstein and Hilbert.)

In view of the importance of these ideas, let us repeat them in slightly different terms. The beauty of Einstein relativity principle is that in reality it implied three unifications. The first, Special Relativity of 1905, (partly) unified Maxwell's electromagnetism with Newton's mechanics by applying the Lorenz transformation to both. The second unification followed a few years later: by recognizing that the Lorenz transformations are really rotations in four-dimensional spacetime if we replace time t by t □ (-1), thus unifying space and time. It appears now almost inevitable to replace Minkowski's (pseudo)Euclidean spacetime by a curved spacetime, in order to interpret curvature by gravitation, thus unifying space, time and gravitation.

One step follows from the other in a logical way, nothing is „put in by hand”. Several other people, Poincare, Lorenz, Minkowsky, Hilbert, Schwarzschild, contributed, but the mastermind was, of course, Einstein.

The next logical step was clear: to unify gravitation and electromagnetism in a generalized *geometrical* model: great scientists such as Hermann Weyl, Eddington, Kaluza, Klein and Einstein himself, tried and failed.

The „dream of a final theory“ (Steven Weinberg) still persists!

By the way, Weyl failed in the most interesting way: his ideas are the base of modern „gauge theories“! As Alfred North Whitehead said, „it is more important that a proposition be interesting than that it be true.”

Section 3. Quantum Theory

In the famous year 1905, Einstein also extended Plank's quantum theory from matter to light, obtaining photons. For this (and another paper on Brownian motion) he got the Nobel Prize; the relativity theory was too controversional to the Nobel Prize Committee.

Nevertheless, all these quantum work was just „epicycles". A unified quantum theory was created around 1925 by Heisenberg, Schroedinger, Dirac and others.

A new „ellipse" (Trick3) replaced the „epicycles" of small ad hoc quantum models as a new „paradigm" in he sense of Kuhn. Quantum theory was and is enormously successful and accurate, just as Relativity is.

Just as the Theory of Relativity was basically the work of *one* person, Einstein, Quantum Theory was founded independently in two completely different mathematical forms by two equally great physicists, Heisenberg and Schrödinger. The miracle was that both mathematical formulations were soon recognized by Dirac, John von Neumann to be equivalent: the underlying Hilbert space may be formulated in two equivalent ways: as a space of functions or a space of infinite-dimensional vectors. A periodic function is equivalent to the infinite vector formed by its Fourier coefficients.

Unfortunately, the two basic theories, relativity and quantum theory, are incompatible. They usually coexist peacefully but steadfastly refuse to be unified. The only rigorous partial unification was Dirac's theory of the electron. Other further attempts are only non-rigorous (though practically very successful) „epicycles": Quantum electrodynamics and quantum gravity [2,3].

Section 4. Modern Approaches

The dream of a final unification persists. There are now an enormous amount of similar but incompatible „epicycle theories".

An important new idea, actually a new paradigm, came from the theory of groups. The „*Eightfold Way*" of Ne'eman and Gell-Mann was very successfully applied to atomic structure (quarks etc.). This is another epicycle.

Gauge theory. The father of this idea is Hermann Weyl. The development is well known. For a unified field theory, this, combined with group theory („dynamic groups") proved the most important mathematical method. Summing up schematically we have: After unsuccessfully gauging the spatio-temporal length element in 1918 (hence the strange name, gauge = Eichung; Eichinvarianz!), Weyl successfully „gauged" the group U(1) to obtain the

gauge theory of electromagnetism, which finally led to gauging the elementary internal symmetry groups U(1) x SU(2) x SU(3) to obtain the „Standard Model" for which Glashow, Salam and Weinberg shared in 1967 a Nobel prize. In fact, Tony Smith pointed out to me that technically they got the Nobel prize already for the unification of U(1) x SU(2), and that the names of 't Hooft and Politzer, Wilczek, and Gross should be added, especially in connection with the SU(3) part. Here, the one-dimensional unitary group U(1) (circular group, a symbol used everywhere, see [6], vol.1), represents *electromagnetism*; the two-dimensional „special unitary" group SU(2) represents the „*weak force*" responsible for radioactive decay, and SU(3) represents the „*strong force*" holding the quarks together.

It should be said that the Standard Model in the present form describes all elementary particles and forces to sufficient accuracy; but theoretically it is considered less elegant than, e.g., General Relativity, because it contains extra features not arising from the theory itself, such as a numerical constant „put in by hand"

A stronger unification is hoped for by Supersymmetry. Much work has been done, but the results seem to be mathematics rather than physics. Jokingly they may be called „epicycles Trick 1".

An incorporation of gravity should lead to a unification of all four basic forces: gravitation, electromagnetism, weak and strong force. This is called „Theory Of Everything", TOE. Schematically we have (or would like to have)

$$\text{TOE} = \text{standard model} + \text{gravity} \rightarrow \text{supersymmetry} + \text{gravity} \rightarrow \text{superstrings}.$$

This is a highly difficult and controversial topic. Many are fascinated by the underlying difficult but highly interesting mathematics. Some of the most brilliant contemporary mathematical physicists work on it. On the other hand we have typical characteristics of epicycles: too many different, mutually incompatible features, one of them being the use of higher-dimensional auxiliary spaces with dimensions ranging from 10 to 27 and more or less. The relation to existing theories or experiments is not clear. They expectations, hopes, and promises rather than solid, mathematically expressed and experimentally verifiable, results.

How many contemporary theoretical elementary particle physicists work in supersymmetry, supergravity and superstrings is seen by looking at the internet! Opinions are divided. The standard reference is [6].

„Supergravity as an answer that looks for a question." (I cannot remember where I read this.)

The very readable popular book on superstrings „The Elegant Universe" [4] looks more like an „elegant anthill."

Richard Feynman said, in Davies and Brown, Superstrings, Cambridge 1988, pp. 194-195):

„... I do feel strongly that this is nonsense! ... I think all this superstring stuff is crazy and is in the wrong direction. ... I don't like it that they're not calculating anything. ... why are the masses of the various particles such as quarks what they are? All these numbers ... have no explanations in these string theories - absolutely none! ...". [5]

Section 5. Present Situation

Let us try to summarize. We have

Fully recognized theories

Classical mechanics (within its limits: c goes to infinity)
Special Relativity (within its limits: no gravitation, flat Minkowski spacetime: pseudo-Euclidean with signature +++-)
General Relativity: curved spacetime, curvature = gravitation
Quantum theory (linear Hilbert space)
Dirac's theory of electron

Dirac theory is a beautiful example of a small, but generally accepted, elegant and powerful unified theory (it predicted the positron!), it introduces new powerful mathematical tools: spinors, Clifford algebras, quaternions. It is the only fully satisfactory (partial) unification of quantum theory and (special) relativity.

The basic internal particle symmetry U(1) x SU(2) x SU(3) as such is generally accepted, as well as relativistic spacetime. The problem is to fuse the two symmetries to get *one* „elegant" and convincing theory.

The *Standard Model* functions very well, but as we remarked, it is criticized for not being sufficiently elegant. On the other hand we have the remark „Elegance is for the tailor" often attributed to the famous Wolfgang Pauli.

We have an

Important heuristic tool:

Mach's Principle: it is only it that gives a full identification inertia = = gravitation and was important for Einstein. Hans-Jürgen Treder is the best expert on it [7].

Recognized mathematical tools:

Between 1920 and 1950 almost all mathematical tools mentioned below have been known (see, e. g., Weyl, Cartan):
Differential Geometry of Riemann space
Hilbert space
Group theory (molecular structure, eightfold way)
Gauge principle (see above)
Clifford algebra (spinors)

And somewhat controversial:

4 Division algebras (there can be only 4):
Real numbers
Complex numbers
Quaternions
Octonions
And nothing more.

The *real numbers* are clear. We should have
Complex numbers ←→ U(1) Electroweak Force
Quaternions ←→ SU(2) Isospin: Protons, Neutrons...
Octonions ←→ SU(3) Strong Force: Quarks etc.

Do we need new mathematical tools no yet discovered, or only a new perspective?

A fascinating historical review, year by year, of these developments from 1900 until today, from Planck to superstrings, can be found at the website http://superstringtheory.com/history/history.history3.html.

Section 6. SU(3) Octonion Theories

They are very interesting alternatives to theorists disappointed by superstring theories. Octonionic theories are rather convincingly supported by a small group of „Platonists":

Geoffrey Dixon
Tony Smith
Corinne. A. Manogue
Jörg Schray
Tevian Dray (USA)
Paolo Budinich (Trieste),...

with generally background support by John Baez (USA). Admitting that they are as little completely elaborated and as controversial as superstring models, I am nevertheless attracted by their strange beauty.

It may be appropriate to say that, in the same way as Gauss represented the complex numbers as 2-vectors, the quaternions may be considered as 4-vectors and the octonions as 8-vectors. The last two vectors are thus „hypercomplex numbers" admitting a product which is zero only if at least one factor is zero. There are *only four types* of such numbers: real and complex numbers, quaternions and octonions. This is a most remarkable mathematical fact, a „Platonic phenomenon" like the well-known five „Platonic Solids".

Thus the use of octonions is potentially capable of limiting the „particle zoo" (number 8!) and avoiding the doubling of the number of particles of supersymmmetry. To Platonists this is, of course, a rather striking advantage,

(Concerning Freudenthal, who did some very advanced work on octonions, I recently mentioned him to Prof. Treder. Of course he knew all about him. As with Plato-Parmenides, I found that he and I are thinking along similar lines, with due regard to his incomparable intellectual superiority. He shares with me the dialectic thinking of Plato.)

Related to the octonions there are the famous Exceptional Algebras (the algebras corresponding to the 5 (no more!) Exceptional Groups) discovered mainly by Elie Cartan: G2, F4, E6, E7, and E8. Another Platonic phenonenon, although these groups have nothing in common with the well known Platonic solids except the total number 5. (Groups and algebras are related by exponentiation; groups are more fundamental but their algebras, being linear, are more manageable.) Dixon uses G2, the (algebra corresponding to the) automorphism group of the octonions.

Smith uses E6 = SL(3, octonions) belonging to a Jordan algebra (named after the famous quantum physicist Pascual Jordan!) Manogue, Dray and Budinich use Cartan spinors (octonions) and Dirac's equation in ten–dimensional space, then „dimensional reduction" from 10- to 4-dimensional space time. The problem attacked by Dixon and Smith, is to incorporate the complete internal symmetry U(1) x SU(2) x SU(3) [8].

Let me try to give my own recent thinking on this problem, which is heavily based on the contributions of Baez and Smith in the Google discussion group mentioned in [8].

Please skip the rest of this section if you wish.

As we have seen in sec. 5 of the present paper, the idea to relate the Cartan all groups through the omnipresent octonions to SU(3), which is connected to

quarks, in much the same way as the quaternions are connected to the electrons via Dirac's theory.

Dixon may be said to start from a 10-dimensional auxiliary space O(9,1) to get Minkowski spacetime O(3,1) (O denotes „orthogonal", and (3,1) means „3 pluses and 1 minus", or „signature" +++-, corresponding to the well-known Minkowski metric (the light velocity is put = 1)

$$ds^2 = + dx^2 + dy^2 + dz^2 – dt^2$$

by „dimensional reduction" by means of a „projection operator". (I apologize for the mathematical jargon and for cheating for the sake of simplification.) (The missing 6 = 9 - 3 dimensions are not lost but give us the „*inner symmetries*" of the Standard Model U(1) x SU(2) x SU(3) mentioned in section 5.)

The essential and beautiful feature is that the initial auxiliary space

$$O(9,1) = SL(2, O),$$

denoting simply a 2 x 2 matrix of determinant 1 whose elements are octonions O. Believe me, this is one of the most beautiful formulas which I know. (I suppose that Plato would be equally delighted.)

T. Smith may be said to start from an equally beautiful formula of the same structure:

$$SL(3, O) = E6$$

where we now have a 3 x 3 matrix of octonions, and E6 denotes one of Cartan's famous Exceptional Algebras, see above. Smith also get the Standard Model (and much more).

Let me only mention that T. Dray and C. A. Manogue start from Dirac's equation formulated in terms of octonions, and P. Budinich uses Cartan spinors in 10 dimensions, all very simple (conceptually!), very clean and very „Platonic" starting points. For references see the Internet.

Section 7. Conclusions

At the beginning of the paper we have seen that, around 1900, there were many epicycles of theories piled up on each other. After the great unification of Einstein and Heisenberg, only two great theories were left: Relativity and Quantum Theory.

For Unification, a completely new idea (view point, „paradigm") is necessary.

All the ingredients were in principle already available, but a change of perspective was required.

Now, 100 years later, we again have even more epicyclical theories: supersymmetry, supergravity, superstrings, spinors and twistors [9], the „eightfold way" of internal symmetries, quaternions and octonions, etc. A generally accepted elegant and basically simple unification of particle forces (electromagnetism, weak and strong force), and hopefully gravitation, in the future is very likely. A change of perspective may do it, but a new Einstein may be necessary to bring about a complete new paradigm in the sense of Kuhn, replacing these epicycles.

The 4th force, gravity, is most problematic because gravitation is intrinsic to spacetime, being is curvature, and General Relativity is such a perfect theory.

All attempts on supersymmetry, supergravity, superstrings and octonionic models deserve our sincere admiration as superb achievements of the human mind. Let us hope that one day (*not too far from 2000*...) the „elegant anthill" will become the „elegant universe" that we all are dreaming of.

Acknowledgment. The octonionic theories were brought to the author's attention a year ago by the enthusiasm of Paolo Budinich. The present paper benefited substantially by the authoritative comments of Tony Smith on a preliminary version of the manuscript.

References

[1] T. S. Kuhn, „*The Structure of Scientific Revolutions*", 2nd ed. Chicago Univ. Press, 1970; see also H. Moritz, „*Science, Mind and the Universe*", Wichmann, Heidelberg 1995, sec. 3-10.

[2] Yu. Ne'eman (ed), „*To Fulfil a Vision*" (Jerusalem Einstein Centennial Symposium), Addison-Wesley, 1981. Even General Relativity can be formulated as a gauge theory; cf. Yu. Ne'eman, „*Gauged and Affine Quantum Gravity*", in this book.

[3] S. W. Hawking and W. Israel (eds.), „*General Relativity: An Einstein Centenary Survey*, Cambridge Univ. Press, 1979; H.-H. von Borzeszkowski and H.-J. Treder, The Meaning of Quantum Gravity, Reidel, Dordrecht, 1987.

[4] B.Greene, *„The Elegant Universe",* Vintage, 2000 (very optimistic).

[5] quoted from Tony Smith's homepage, which is also freely downloadable as a compact Pdf-Book „TonySwebBook2.pdf", 2003.

[6] The standard work is S. Weinberg, „*The Quantum Theory of Fields*", Vols. I to III, Cambridge Univ. Press, 1995-2002. For groups and supergroups I found very helpful: L. Frappat, A. Sciarino and P. Sorba, *Dictionary on Lie Algebras and Superalgebras,* Academic Press, London 2000.

[7] H.-J. Treder, *Die Relativität der Trägheit*, Akademie-Verlag, Berlin,1972; H-.J. Treder, H.-H. von Borzeszowski, Alwyn van der Merwe, W. Yourgrau, *Fundamental Principles of General Relativity*, Plenum Press, New York, 1980.

[8] See John Baez, „*The Octonions*", 2001, Internet; Dixon, *Division Algebras*, Kluwer, 1994 (in addition, he has an interesting homepage at www.7stones.com); Tony Smith's homepage (Internet) and WebBook [5]. The mathematics is very difficult, there is no „Koenigsweg" (royal road) to is, but you can use P. Lounesto, Clifford Algebras and Spinors, Cambridge Univ.Press, 2001 and Baez 2001 for an introduction, supplemented by the *„Dictionary"* mentioned in [6]. (Concerning „*dimensional reduction*", there is a paper with this title by Manogue and Dray (arXiv:hep-th/9807044 v1 6 Jul 1998), see also[5], p. 351 and the Thread „Dimensional reduction for elementary particles" in Google's Newsgroup sci.physics.research 2003). For a comparison of Dixon's and Smith's models see this Thread and [5], pp. 293 and 1717. You can find almost all information on contemporary advanced topics in the internet using your favorite search machine!

[9] R. Penrose and W. Rindler, *Spinors and Spacetime*, Vol. 2, Cambridge Univ. Press, 1986.

Anmerkung Hans-Jürgen Treders zu vorstehendem Beitrag

Die Epizyklen der Weltsysteme werden gemäß Apolonios von Perge (262–190 v. Chr.) durch die Transformationen der Radius-Vektoren r_A kinematisch gemäß $\bar{r}_A = r_A + \dot{r}_A t$, mit den Geschwindigkeitsvektoren $\dot{r}_A = v_A(t) = dr_A/dt$ ineinander überführt. Apollonius zeigte damit die „kinematische Äquivalenz" des geozentrischen Weltsystems gemäß Eudoxos (408–320 v. Chr.) und des heliozentrischen Weltsystems des Aristarch von Samos.

Die peripatetische Physik des Aristoteles verlangt nun $dr_A/dt = 0$ (für die „sublunare Sphäre"). Damit begründete K. Ptolemaios (~100–170) physikalisch das geozentrische Weltsystem.

Die klassische Mechanik verlangt einen „absoluten Raum" im Sinne Newtons, d.h. „absolute Beschleunigungen". Dies reduziert die kinematische Gruppe auf die Galilei-Gruppe $\bar{r}_A = r_A + v_A t$ mit $v_A = \text{const}$. Machs Prinzip der „Relativität der Trägheit" (und Einsteins „Allgemeine Relativität") verlangen dagegen wieder die vollständige kinematische Gruppe. Diese wird nun als Poincaré-Gruppe in die Gravo-Dynamik eingeführt (Mach-Einstein-Doktrin).

Vergleiche H.-J. Treder: Eine analytische Fassung der Mach-Einsteinschen Relativität der Trägheit. Symposia Mathematica Bologna 1973 und H.-J. Treder: Über die Prinzipien der Dynamik von Einstein, Hertz, Mach und Poincaré. Berlin Akademie-Verlag, 1974.