Operational Geodesy, Rank Deficiencies and Geodetic Heights

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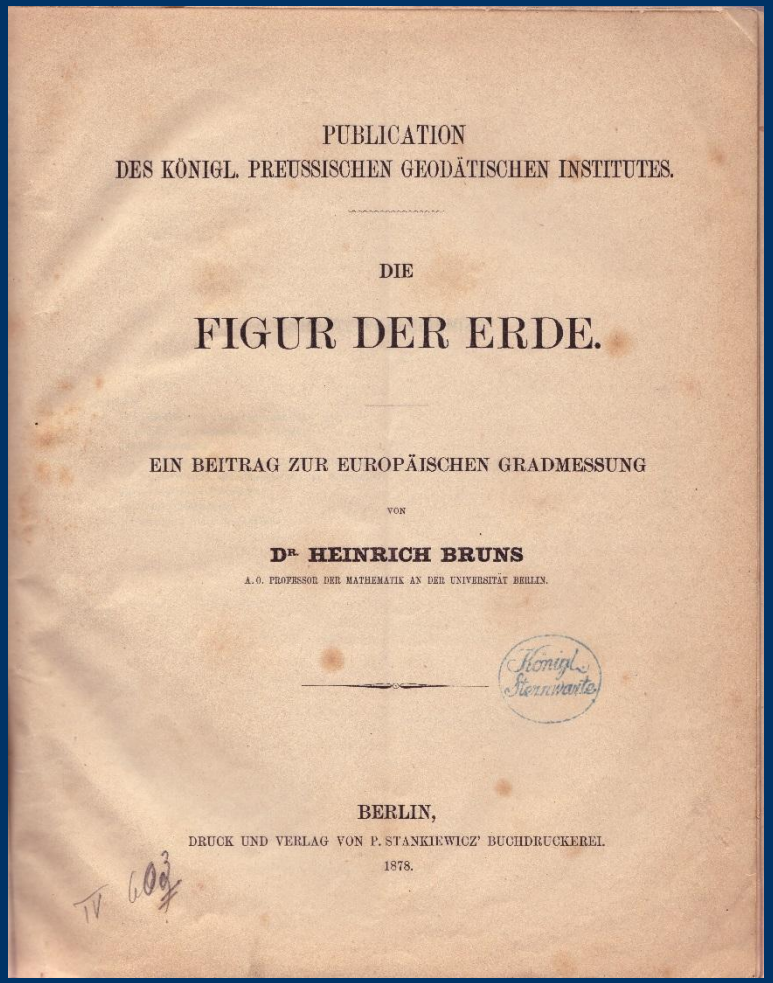
Kolloquium der Leibniz-Sozietät am 13.2.2015
„Geodäsie – Mathematik – Physik – Geophysik“
zum 75. Geburtstag von Prof. Dr.-Ing. habil. Dr. h.c. mult. Erik W. Grafarend

Bruns‘ Figur der Erde

Heinrich Bruns (1848 -1919)



PUBLICATION
DES KÖNIGL. PREUSSISCHEN GEODÄTISCHEN INSTITUTES.

DIE

FIGUR DER ERDE.

EIN BEITRAG ZUR EUROPÄISCHEN GRADMESSUNG

VON

DR. HEINRICH BRUNS

A. O. PROFESSOR DER MATHEMATIK AN DER UNIVERSITÄT BERLIN.

Königl. Sternwarte

BERLIN,

DRUCK UND VERLAG VON P. STANKIEWICZ' BUCHDRUCKEREI.

1878.

IV 6.03

Bruns‘ Figur der Erde

Classification of geodetic observables into

- astronomical positioning (polar angle, longitude, azimuth)
- triangulation (horizontal angles, baseline)
- trigonometric levelling (zenith angles)
- geometric levelling
- gravimetry

Quality of geodetic observations (e.g. zenith angles and refraction)

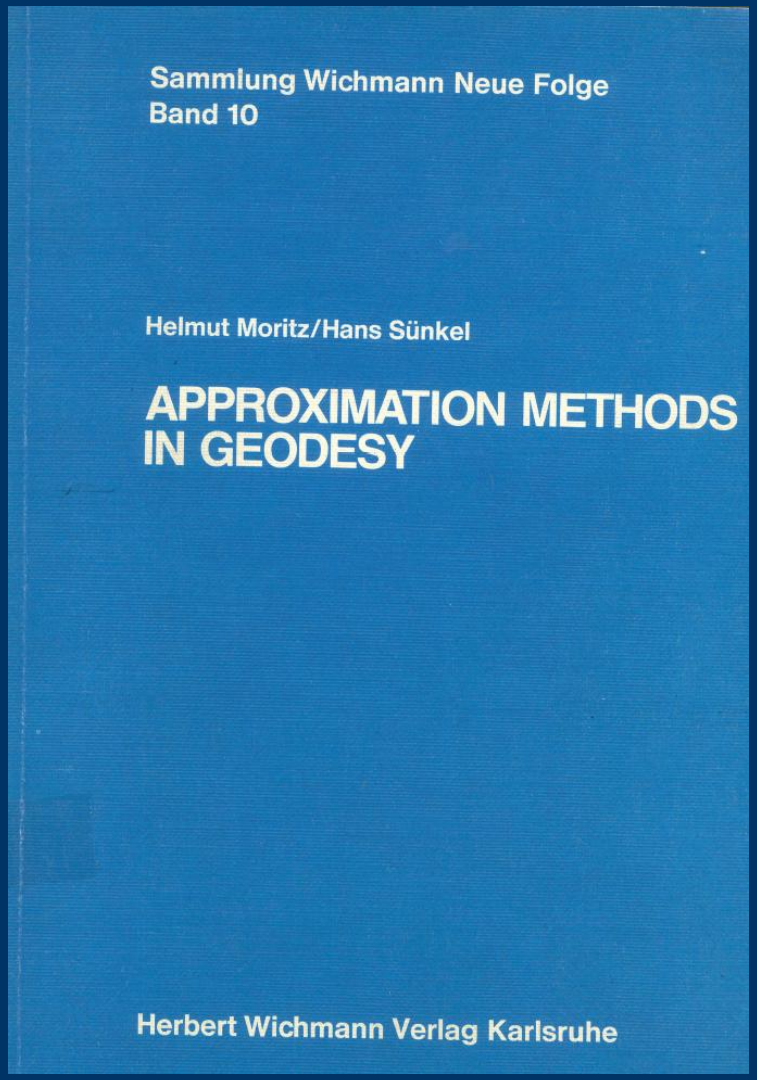
Estimable quantities:
Threedimensional polyhedron (Bruns‘ polyhedron) , including ΔW and g at vertices of polyhedron and geoid
If polyhedron would be global (no oceans):
in addition W and the orientation of the geoid w.r.t. the spin axis

Bruns discusses:

- weakness of zenith angles (refraction)
- geoid versus sea surface (atmospheric pressure, tides, circulation, wind stress)

Erik Grafarends Operational Geodesy

published in 1978 (100 years after Bruns‘ „Figur der Erde“) in:



Lectures delivered at the
Second International Summer School
in the Mountains on
Mathematical Methods in Physical Geodesy
Ramsau, Austria, 23.8 – 2.9.1977

Erik Grafarends Operational Geodesy

Concept of operational geodesy:
Theoretical framework for the determination of the figure of the earth and the exterior (and interior) gravity field
figure of the earth expressed in finite element approximation
--- satellite geodesy excluded ---
Starting point: geodetic observables,
their contribution to the solution of the geodetic boundary value problem
Elements of operational geodesy:

- analysis of all geodetic observables (geometry, gravity, others)
- formalism of the linearization process (also in terms of notation)
- hierarchy of reference frames
- classification of linear(ized) models in terms of stochastic/deterministic properties
- rank defect analysis and geodetic datum
- various linear estimators

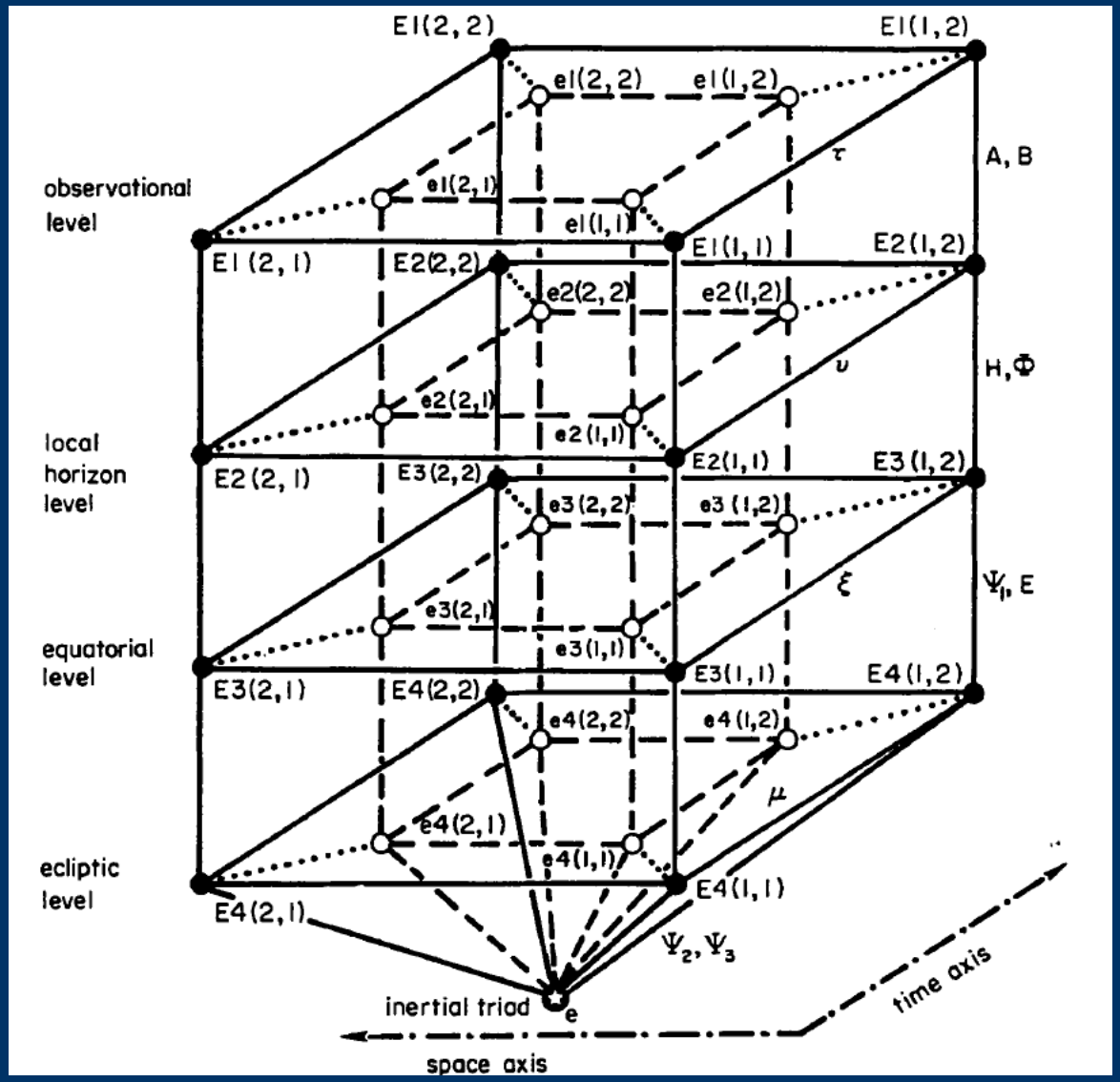
Inspiration and foundation for later work

spacelike geodetic observational equations

type of observations	- point function	X˙,Y˙,Z˙ spacelike unknowns	ellipsoidal datum unknowns	type of additional unknowns	total number of unknowns
astronomic longitude Λ	1	3	10	$\delta\lambda$	14
astronomic latitude Φ	1	3	10	$\delta\phi$	14
absolute gravity Γ	1	3	10	$\delta\gamma$	14
scalar potential differences ΔW	2	6	10	$\delta w_i, \delta w_j$	18
astronomic azimuth A	2	6	10	$\delta\lambda, \delta\phi$	18
horizon distance B	2	6	10	$\delta\lambda, \delta\phi$	18
horizontal direction T	2	6	10	$\delta\lambda, \delta\phi, \sum$	19
distance S	2	6	—	—	6
horizontal angle ΔT	3	9	10	$\delta\lambda, \delta\phi$	21
distance ratios	3	9	—	—	9
positional angles	3	9	—	—	9

Grafarend EW, Operational Geodesy, Table 1, p.248, 1978

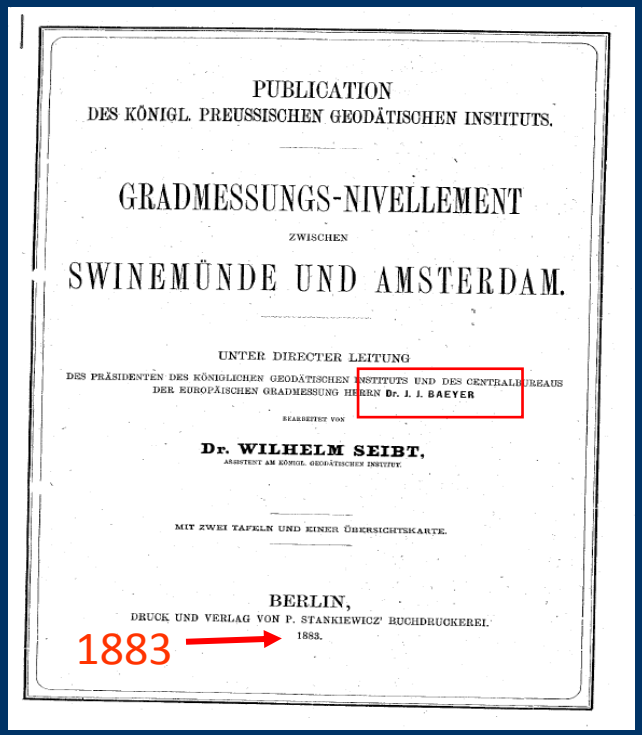
concepts for reference frames in geodesy and geodynamics



Grafarend EW, Il Mueller,H Papo & B Richter, Bull. Geod, 1978

geodetic (spirit) levelling

- geodetic levelling = most elementary form of network (1D)
- measured are potential differences (height increments x gravity)
- datum arbitrarily chosen, usually mean sea level at tide gauge



PUBLICATION
DES KÖNIGL. PREUSSISCHEN GEODÄTISCHEN INSTITUTS.

GRADMESSUNGS-NIVELLEMENT
ZWISCHEN
SWINEMÜNDE UND AMSTERDAM.

UNTER DIRECTER LEITUNG
DES PRÄSIDENTEN DES KÖNIGLICHEN GEODÄTISCHEN INSTITUTS UND DES CENTRALBUREAUS
DER EUROPÄISCHEN GRADMESSUNG HERRN Dr. J. J. BAEYER

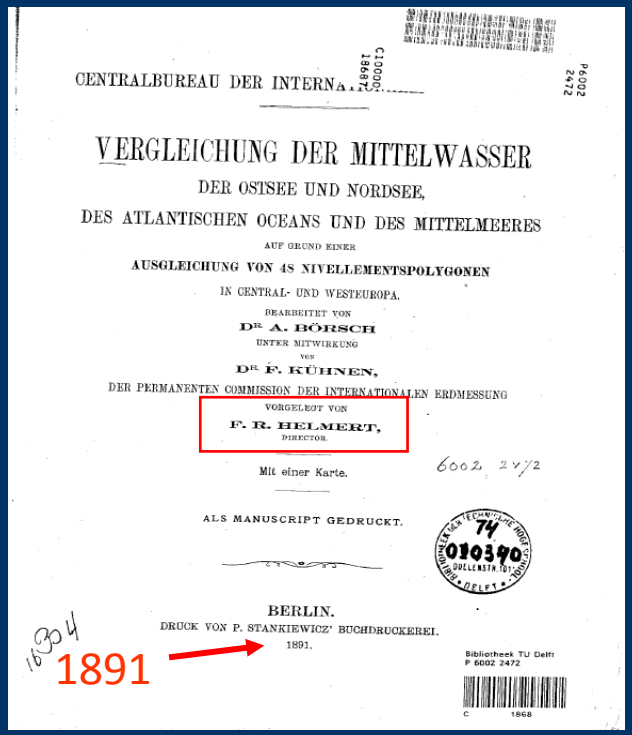
BEARBEITET VON

Dr. WILHELM SEIBT,
ASSISTENT AM KÖNIGL. GEODÄTISCHEN INSTITUT.

MIT ZWEI TAFELN UND EINER ÜBERSICHTSKARTE.

BERLIN,
DRUCK UND VERLAG VON P. STANKIEWICZ' BUCHDRUCKEREI.
1883.

1883



CENTRALBUREAU DER INTERNA...

VERGLEICHUNG DER MITTELWASSER
DER OSTSEE UND NORDSEE,
DES ATLANTISCHEN OCEANS UND DES MITTELMEERES
AUF GRUND EINER
AUSGLEICHUNG VON 48 NIVELLEMENTSPOLYGONEN
IN CENTRAL- UND WESTEUROPA.
BEARBEITET VON
DR A. BÖRSCH
UNTER MITWIRKUNG
VON
DR F. KÜHNEN,
DER PERMANENTEN COMMISSION DER INTERNATIONALEN ERDMESSUNG
VORGELEGT VON
F. R. HELMERT,
DIRECTOR.

Mit einer Karte.

ALS MANUSCRIPT GEDRUCKT.

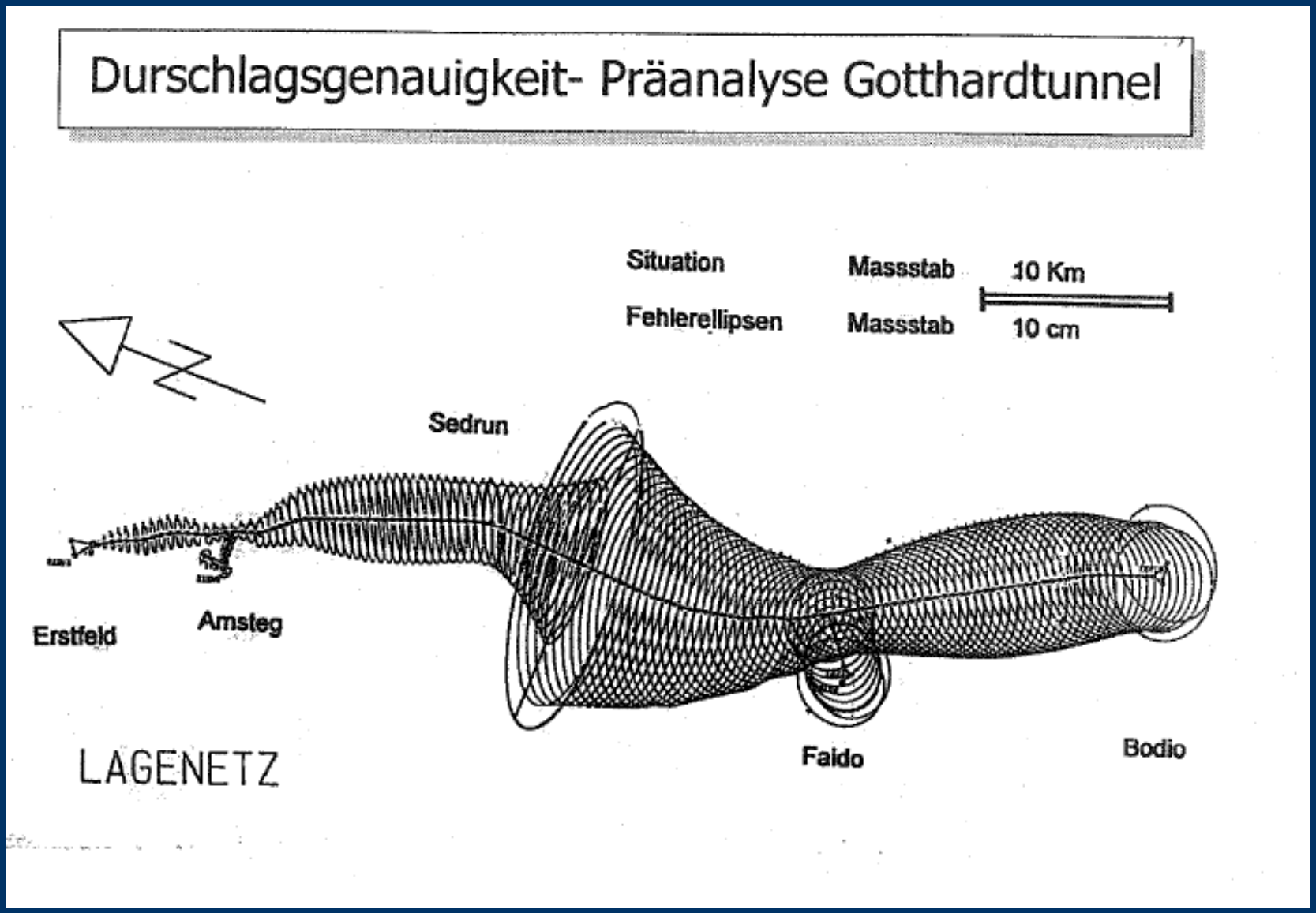
BERLIN.
DRUCK VON P. STANKIEWICZ' BUCHDRUCKEREI.
1891.

1891

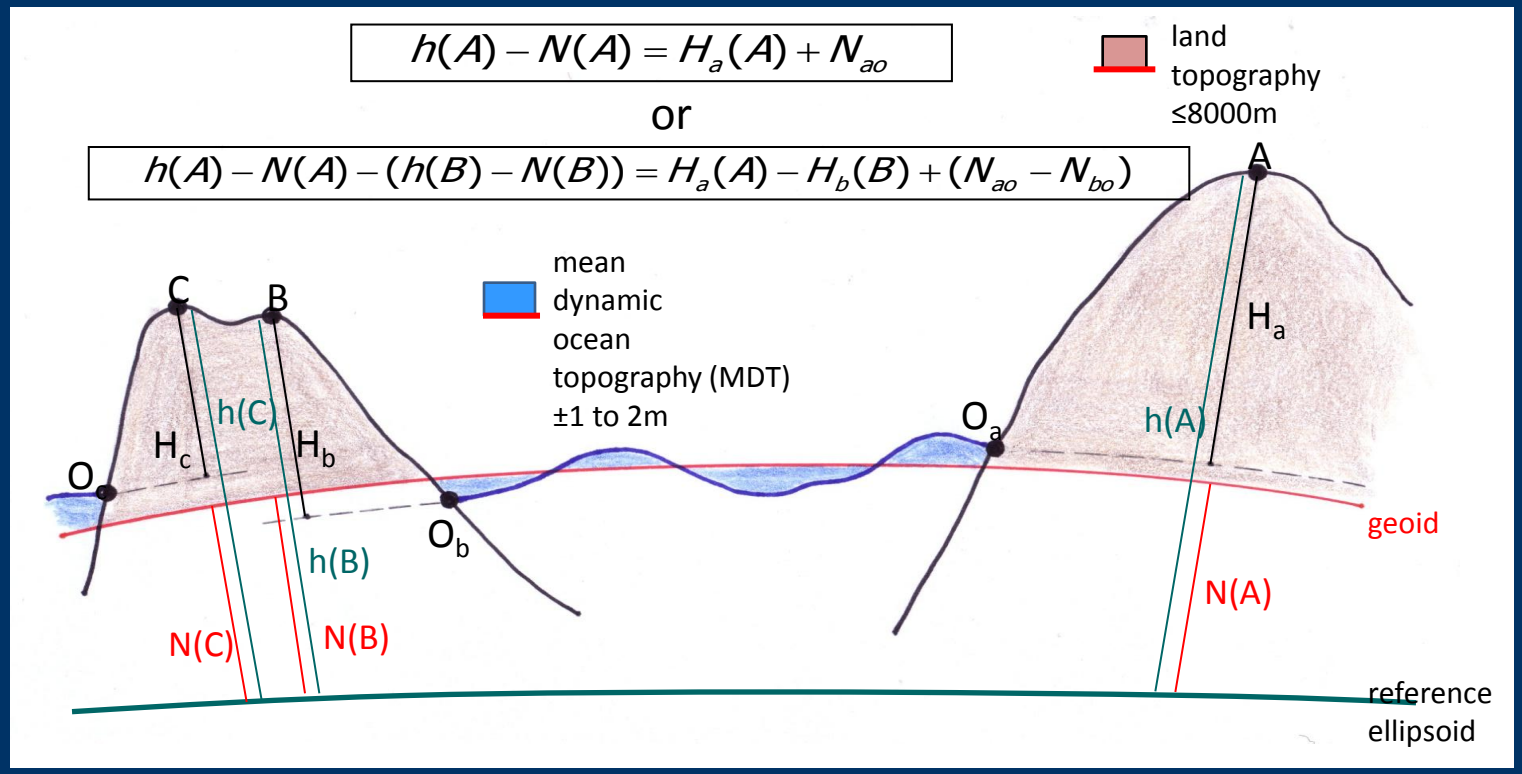
geodetic datum

- Datum parameters are the minimum information to be added to a geodetic network, in order to make coordinates estimable quantities (eliminates rank deficiency, fills null space)
- Their choice is arbitrary as long as they eliminate the rank deficiency. No choice is better than another one; however, one choice may feel more natural than another one
- Datum parameters are not stochastic. Error ellipses shrink to zero at the datum point
- Fixing more datum parameters than necessary will result in an over-constraint (fixing more than one height in a levelling network leads to a deformation)

geodetic datum



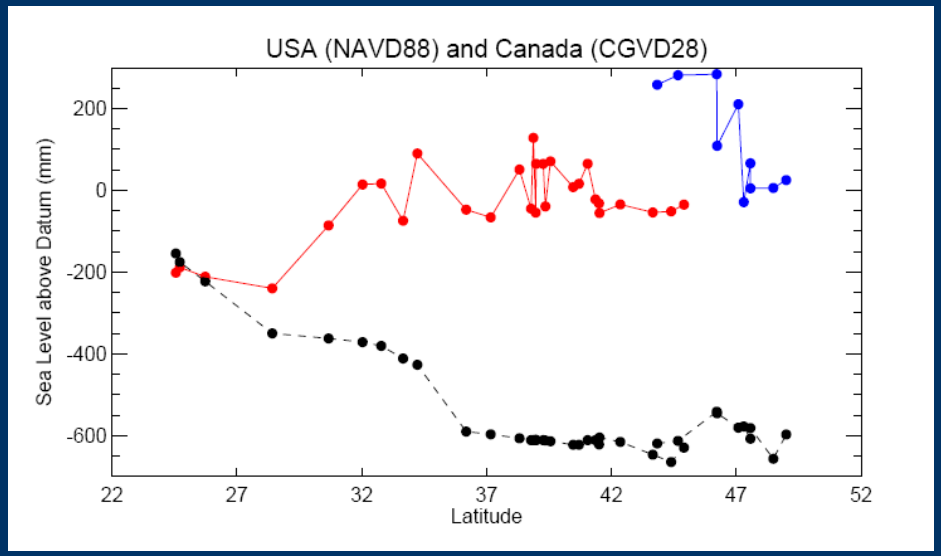
principle of height system unification



- apply principle of "GPS-levelling", i.e. *h* from GNSS or SLR and *N* from GOCE
- take into account short-wavelength geoid part (omission part)
- identify existing height off-sets
- create unified global height system

GPS heights versus geodetic (spirit) levelling

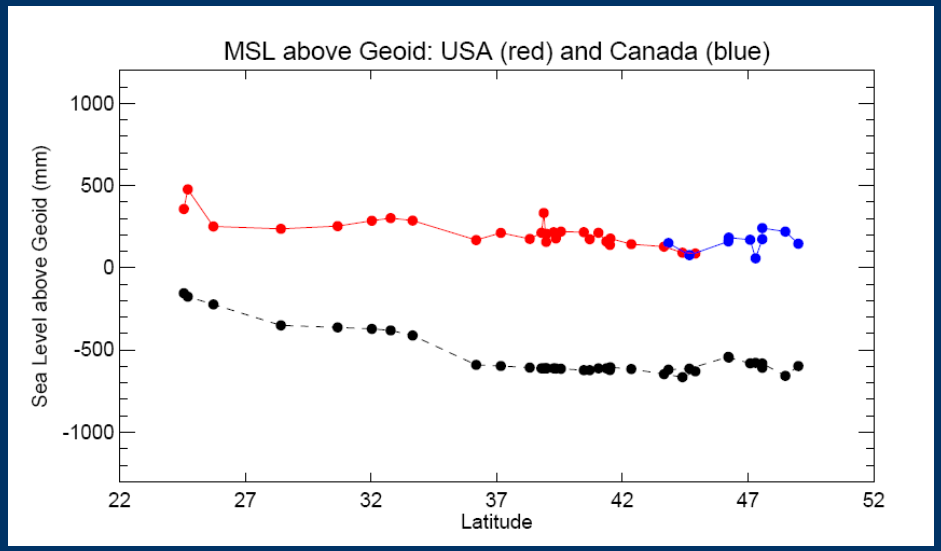
a historical controversy (Sturges, W, 1974, *JGR* versus Fischer, I, 1975, *Bulletin Geodesique*): about the correct tilt of sea level along the east coast of North America



before:

spirit levelling
connecting the tide gauges of USA (red) and Canada (blue)

numerical ocean model (black)



after:

GPS-heights minus GRACE/GOCE geoid
(complemented by short wavelength geoid contribution from EGM2008)

numerical ocean model (black)

Woodworth PL, CW Hughes RJ Bingham & T Gruber, *J Geod Sci*, 2012

determination of W

- Only potential differences ΔW are measurable
- Why can we determine W ?
- Role of regularity condition of GBVP
- W_0 and the geoid

Bruns H.: Figur der Erde 1878, S. 46
Heiskanen W., H. Moritz: Physical Geodesy, 1967, S. 103 „Determination of N_0“
Sacerdote F., F. Sansò: W_0 - A story of the height datum problem, Hannover, 2001
Hipkin R.G.: Is there a need for a geodetic datum 2000? Discussion of „H-M proposition“, 2001

Burša M.,Radej K., Šima Z., True, S.A., Vatrt V., Studia Geoph. Geod., 1997
Burša M., Kenyon S., Kouba J., Šima Z., Vatrt V., Vitek V., Vojtišková, J Geod., 2007
Grafarend E.W., Ardalan A.A., J Geod, 1997
Ardalan A.A., Grafarend E.W., Finnish Geod Inst, 1999

determination of W

AE Gill (1982) Atmosphere-Ocean Dynamics, p.46:

> "If the sea were at rest, its surface would coincide with the geopotential surface."

W_0 is the potential value of the geoid; but what is an operational definition of the geoid?

- level surface through a chosen reference point
- Mean of selected tide gauges
- Mean of altimetric mean sea surface

Geoid definition:

- easy at an uncertainty level of above 10cm
- complex at a level of between 1cm and 10cm
- almost impossible if an accuracy is required of better than 1cm

Mather R.S. (1978): The role of the geoid in four-dimensional geodesy, Marine Geodesy, 1:3, 217-252

Moritz H. (1980): Advanced Physical Geodesy, Wichmann

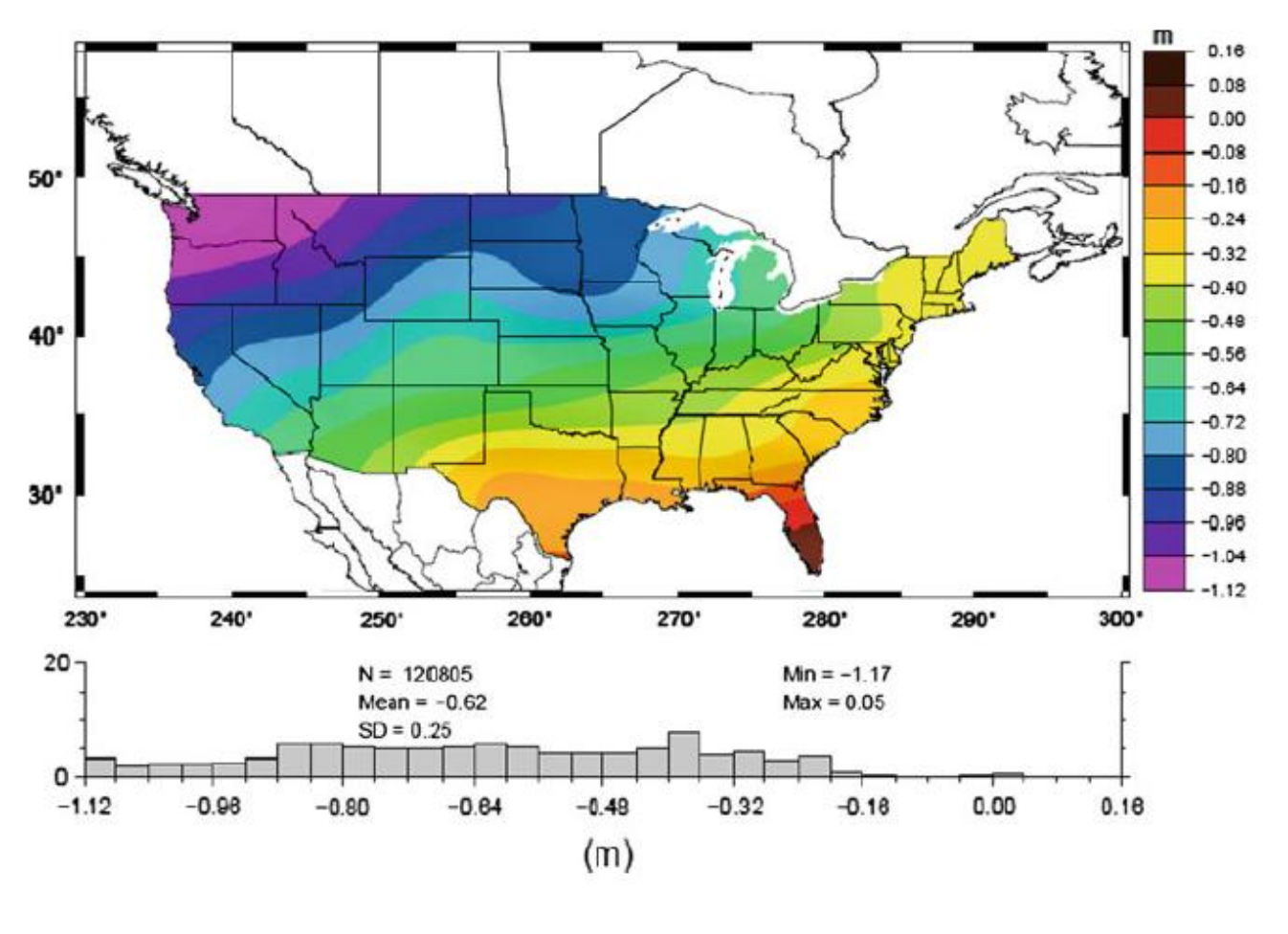
Erik Grafarends Operational Geodesy

Erik Grafarends gift to geodesy and its adjacent disciplines:

- new fundamental concepts
- structure
- rigor
- elegance
- inspiration
- hospitality
- guidance
- prestige
- …

„What would be more operational?“

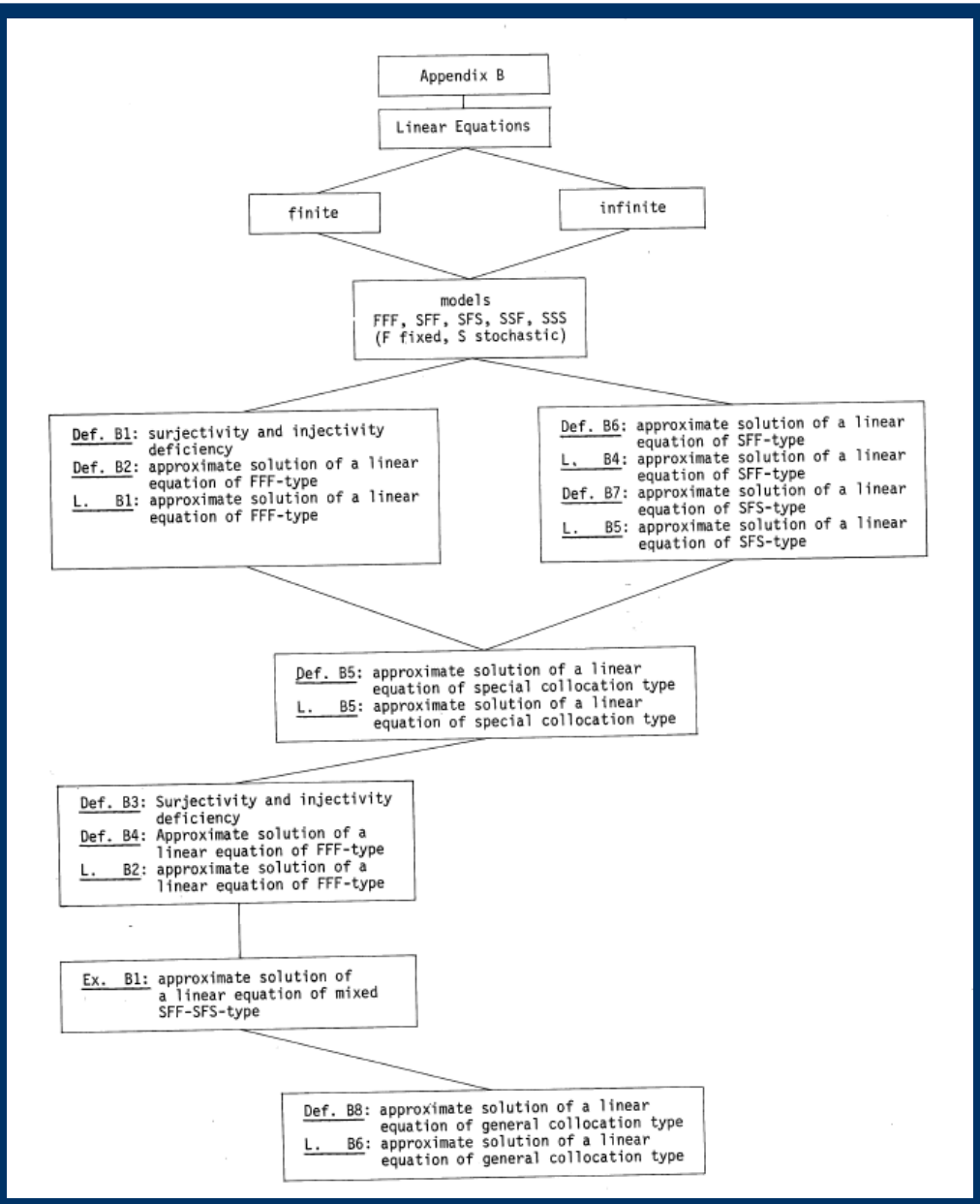
GPS heights versus geodetic (spirit) levelling



long-wavelength erros of the US vertical network

Wang Y.M., Saleh J., Roman D.R. J Geod 2012

classification of linear geodetic estimation models



Grafarend, Operational Geodesy, Table 2, p.253, 1978

type of observations	- point function	X',Y',Z' spacelike unknowns	ellipsoidal datum unknowns	type of additional unknowns	total number of unknowns
astronomic longitude Λ	1	3	10	$\delta\lambda$	14
astronomic latitude Φ	1	3	10	$\delta\phi$	14
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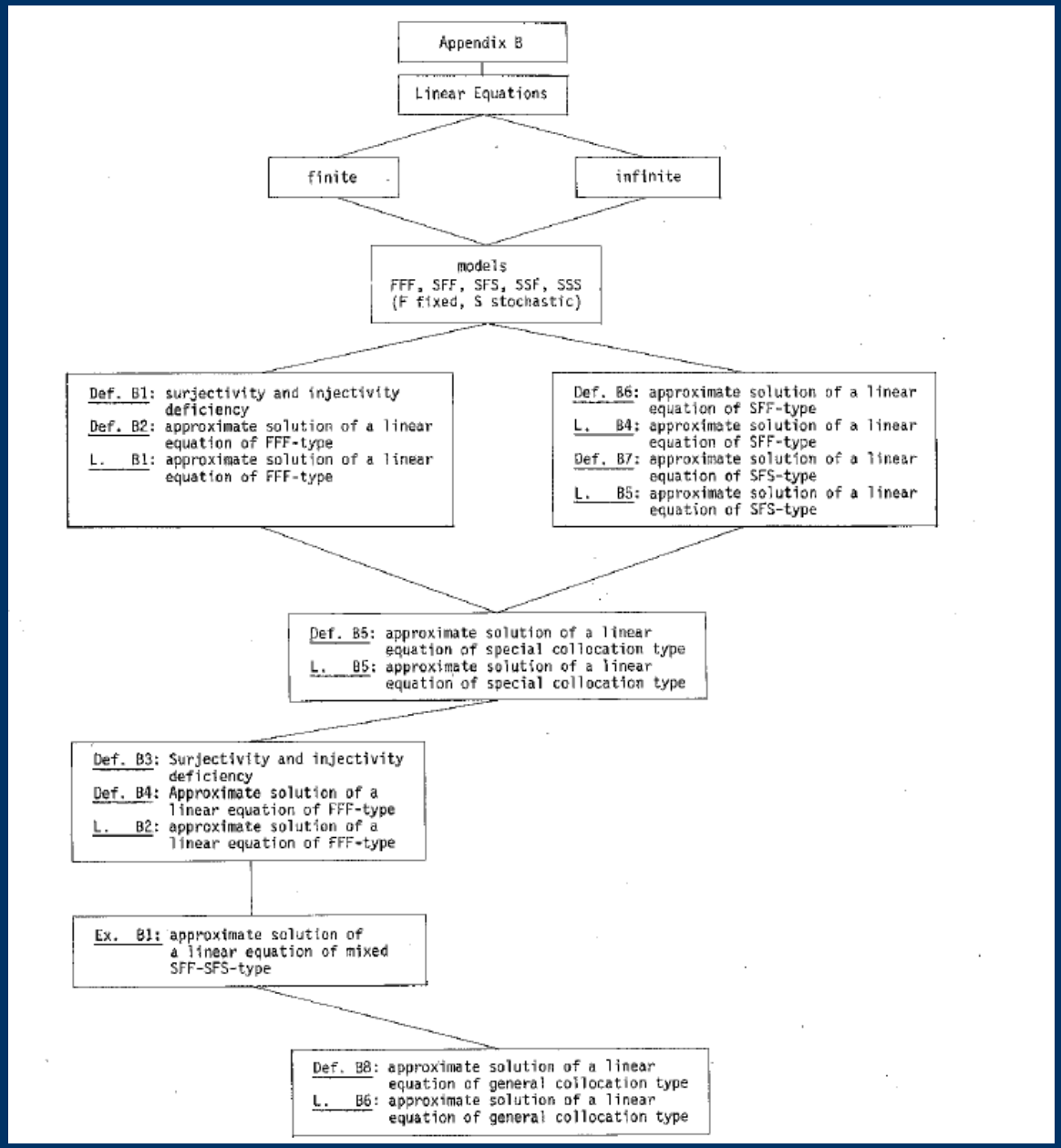
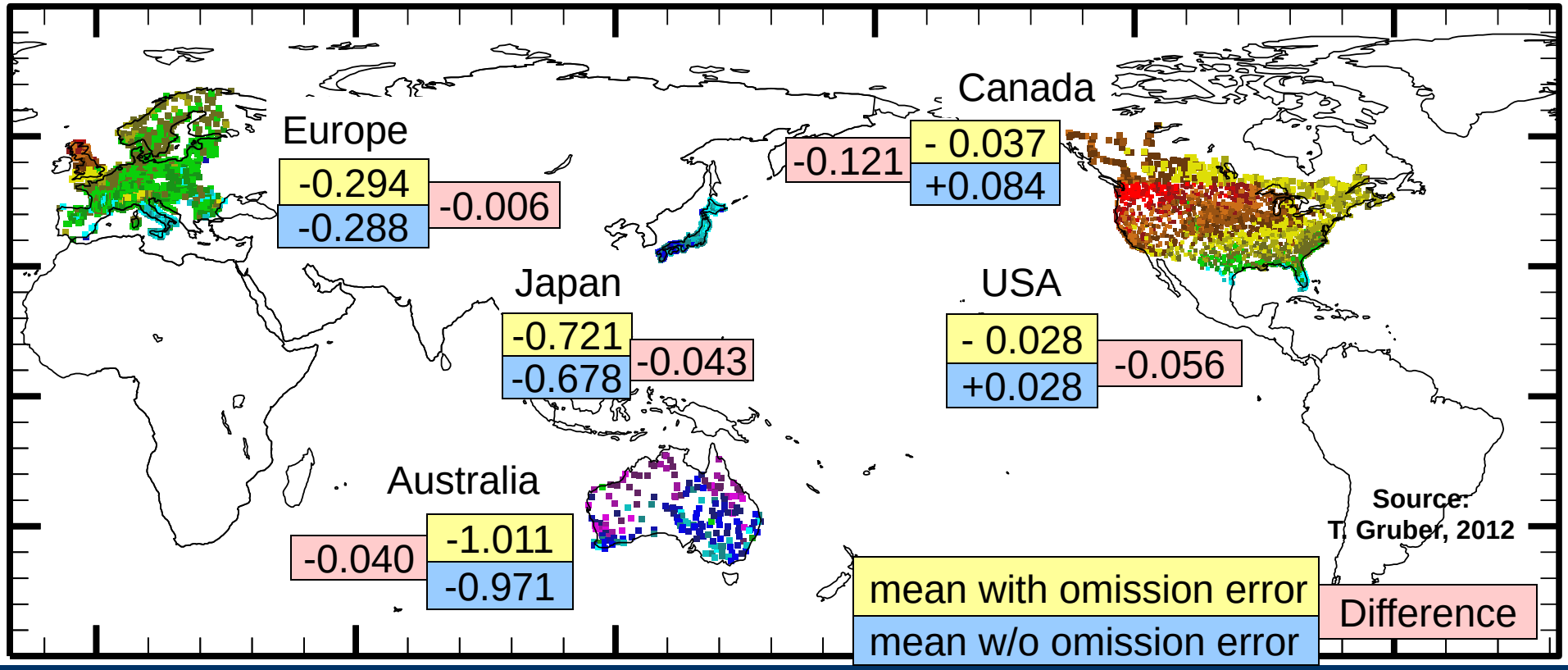

Appendix B
Linear Equations
finite
infinite
models
FFF, SFF, SFS, SSF, SSS
(F fixed, S stochastic)
Def. B1: surjectivity and injectivity deficiency
Def. B2: approximate solution of a linear equation of FFF-type
L. B1: approximate solution of a linear equation of FFF-type
Def. B6: approximate solution of a linear equation of SFF-type
L. B4: approximate solution of a linear equation of SFF-type
Def. B7: approximate solution of a linear equation of SFS-type
L. B5: approximate solution of a linear equation of SFS-type
Def. B5: approximate solution of a linear equation of special collocation type
L. B5: approximate solution of a linear equation of special collocation type
Def. B3: Surjectivity and injectivity deficiency
Def. B4: Approximate solution of a linear equation of FFF-type
L. B2: approximate solution of a linear equation of FFF-type
Ex. B1: approximate solution of a linear equation of mixed SFF-SFS-type
Def. B8: approximate solution of a linear equation of general collocation type
L. B6: approximate solution of a linear equation of general collocation type

theoretical results

Preliminary tests show:

Modeling of the omission part with EGM2008 in well surveyed countries leaves an uncertainty of below 10cm (see: Gruber)



TIM3 d/o=200