Operational Geodesy, Rank Deficiencies and Geodetic Heights

Reiner Rummel
Institut für Astronomische und Physikalische Geodäsie
Technische Universität München
rummel@bv.tum.de

Kolloquium der Leibniz-Sozietät am 13.2.2015
„Geodäsie – Mathematik – Physik – Geophysik“
zum 75. Geburtstag von Prof. Dr.-Ing. habil. Dr. h.c. mult. Erik W. Grafarend
Bruns’ Figur der Erde

Heinrich Bruns (1848 -1919)
Classification of geodetic observables into
- astronomical positioning (polar angle, longitude, azimuth)
- triangulation (horizontal angles, baseline)
- trigonometric levelling (zenith angles)
- geometric levelling
- gravimetry

Quality of geodetic observations (e.g. zenith angles and refraction)

Estimable quantities:
Threedimensional polyhedron (Bruns‘ polyhedron), including $\Delta W$ and $g$
at vertices of polyhedron and geoid
If polyhedron would be global (no oceans):
in addition $W$ and the orientation of the geoid w.r.t. the spin axis

Bruns discusses:
- weakness of zenith angles (refraction)
- geoid versus sea surface (atmospheric pressure, tides, circulation, wind stress)
Erik Grafarends Operational Geodesy

published in 1978 (100 years after Bruns‘ „Figur der Erde“) in:

Lectures delivered at the Second International Summer School in the Mountains on Mathematical Methods in Physical Geodesy Ramsau, Austria, 23.8 – 2.9.1977
Concept of operational geodesy:
Theoretical framework for the determination of the figure of the earth and the exterior (and interior) gravity field figure of the earth expressed in finite element approximation
--- satellite geodesy excluded ---
Starting point: geodetic observables, their contribution to the solution of the geodetic boundary value problem
Elements of operational geodesy:
• analysis of all geodetic observables (geometry, gravity, others)
• formalism of the linearization process (also in terms of notation)
• hierarchy of reference frames
• classification of linear(ized) models in terms of stochastic/deterministic properties
• rank defect analysis and geodetic datum
• various linear estimators
Inspiration and foundation for later work
### Spacelike Geodetic Observational Equations

<table>
<thead>
<tr>
<th>Type of Observations</th>
<th>Point Function</th>
<th>(X', Y', Z') Spacelike Unknowns</th>
<th>Ellipsoidal Datum Unknowns</th>
<th>Type of Additional Unknowns</th>
<th>Total Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astronomic Longitude (\Lambda)</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>(\delta\lambda)</td>
<td>14</td>
</tr>
<tr>
<td>Astronomic Latitude (\phi)</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>(\delta\phi)</td>
<td>14</td>
</tr>
<tr>
<td>Absolute Gravity (\Gamma)</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>(\delta\gamma)</td>
<td>14</td>
</tr>
<tr>
<td>Scalar Potential Differences (\Delta W)</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>(\delta W_i, \delta W_j)</td>
<td>18</td>
</tr>
<tr>
<td>Astronomic Azimuth (A)</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>(\delta\lambda, \delta\phi)</td>
<td>18</td>
</tr>
<tr>
<td>Horizon Distance (B)</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>(\delta\lambda, \delta\phi)</td>
<td>18</td>
</tr>
<tr>
<td>Horizontal Direction (T)</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>(\delta\lambda, \delta\phi, \delta\psi)</td>
<td>19</td>
</tr>
<tr>
<td>Distance (S)</td>
<td>2</td>
<td>6</td>
<td>—</td>
<td>—</td>
<td>6</td>
</tr>
<tr>
<td>Horizontal Angle (\Delta T)</td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>(\delta\lambda, \delta\phi)</td>
<td>21</td>
</tr>
<tr>
<td>Distance Ratios</td>
<td>3</td>
<td>9</td>
<td>—</td>
<td>—</td>
<td>9</td>
</tr>
<tr>
<td>Positional Angles</td>
<td>3</td>
<td>9</td>
<td>—</td>
<td>—</td>
<td>9</td>
</tr>
</tbody>
</table>
concepts for reference frames in geodesy and geodynamics
geodetic (spirit) levelling

- geodetic levelling = most elementary form of network (1D)
- measured are potential differences (height increments x gravity)
- datum arbitrarily chosen, usually mean sea level at tide gauge
Datum parameters are the minimum information to be added to a geodetic network, in order to make coordinates estimable quantities (eliminates rank deficiency, fills null space)

Their choice is arbitrary as long as they eliminate the rank deficiency. No choice is better than another one; however, one choice may feel more natural than another one.

Datum parameters are not stochastic. Error ellipses shrink to zero at the datum point.

Fixing more datum parameters than necessary will result in an over-constraint (fixing more than one height in a levelling network leads to a deformation).
geodetic datum
\[ h(A) - N(A) = H_a(A) + N_{ao} \]

or
\[ h(A) - N(A) - (h(B) - N(B)) = H_a(A) - H_b(B) + (N_{ao} - N_{bo}) \]

- apply principle of “GPS-levelling”, i.e. \( h \) from GNSS or SLR and \( N \) from GOCE
- take into account short-wavelength geoid part (omission part)
- identify existing height off-sets
- create unified global height system
GPS heights versus geodetic (spirit) levelling


**Before:**
- Spirit levelling
- Connecting the tide gauges of USA (red) and Canada (blue)

**Numerical ocean model (black)**

**After:**
- GPS-heights minus GRACE/GOCE geoid
- (Complemented by short wavelength geoid contribution from EGM2008)

**Numerical ocean model (black)**

Woodworth PL, CW Hughes RJ Bingham & T Gruber, J Geod Sci, 2012
determination of W

• Only potential differences $\Delta W$ are measurable
• Why can we determine $W$?
• Role of regularity condition of GBVP
• $W_0$ and the geoid

Bruns H.: Figur der Erde 1878, S. 46
Heiskanen W., H. Moritz: Physical Geodesy, 1967, S. 103 „Determination of $N_0“

Burša M., Kenyon S., Kouba J., Šima Z., Vatrt V., Vitek V., Vojtišková, J Geod., 2007
Grafarend E.W., Ardalan A.A., J Geod, 1997
“If the sea were at rest, its surface would coincide with the geopotential surface.”

$W_0$ is the potential value of the geoid; but what is an operational definition of the geoid?

- level surface through a chosen reference point
- Mean of selected tide gauges
- Mean of altimetric mean sea surface

Geoid definition:

- easy at an uncertainty level of above 10cm
- complex at a level of between 1cm and 10cm
- almost impossible if an accuracy is required of better than 1cm
Erik Grafarends gift to geodesy and its adjacent disciplines:

- new fundamental concepts
- structure
- rigor
- elegance
- inspiration
- hospitality
- guidance
- prestige
- ...

„What would be more operational?“
GPS heights versus geodetic (spirit) levelling

long-wavelength errors of the US vertical network

Wang Y.M., Saleh J., Roman D.R. J Geod 2012
classification of linear geodetic estimation models
<table>
<thead>
<tr>
<th>Type of Observations</th>
<th>- Point Function</th>
<th>X', Y', Z' Spacelike Unknowns</th>
<th>Ellipsoidal Datum Unknowns</th>
<th>Type of Additional Unknowns</th>
<th>Total Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Astronomic Longitude λ</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>δλ</td>
<td>14</td>
</tr>
<tr>
<td>Astronomic Latitude φ</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>δφ</td>
<td>14</td>
</tr>
<tr>
<td>Absolute Gravity γ</td>
<td>1</td>
<td>3</td>
<td>10</td>
<td>δγ</td>
<td>14</td>
</tr>
<tr>
<td>Scalar Potential Differences ΔW</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>Δw_i, Δw_j</td>
<td>18</td>
</tr>
<tr>
<td>Astronomic Azimuth A</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>δλ, δφ</td>
<td>18</td>
</tr>
<tr>
<td>Horizon Distance B</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>δλ, δφ</td>
<td>18</td>
</tr>
<tr>
<td>Horizontal Direction T</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>δλ, δφ, Σ</td>
<td>19</td>
</tr>
<tr>
<td>Distance S</td>
<td>2</td>
<td>6</td>
<td>—</td>
<td>—</td>
<td>6</td>
</tr>
<tr>
<td>Horizontal Angle ΔT</td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>δλ, δφ</td>
<td>21</td>
</tr>
<tr>
<td>Distance Ratios</td>
<td>3</td>
<td>9</td>
<td>—</td>
<td>—</td>
<td>9</td>
</tr>
<tr>
<td>Positional Angles</td>
<td>3</td>
<td>9</td>
<td>—</td>
<td>—</td>
<td>9</td>
</tr>
</tbody>
</table>
Preliminary tests show:
Modeling of the omission part with EGM2008 in well surveyed countires leaves an uncertainty of below 10cm (see: Gruber)